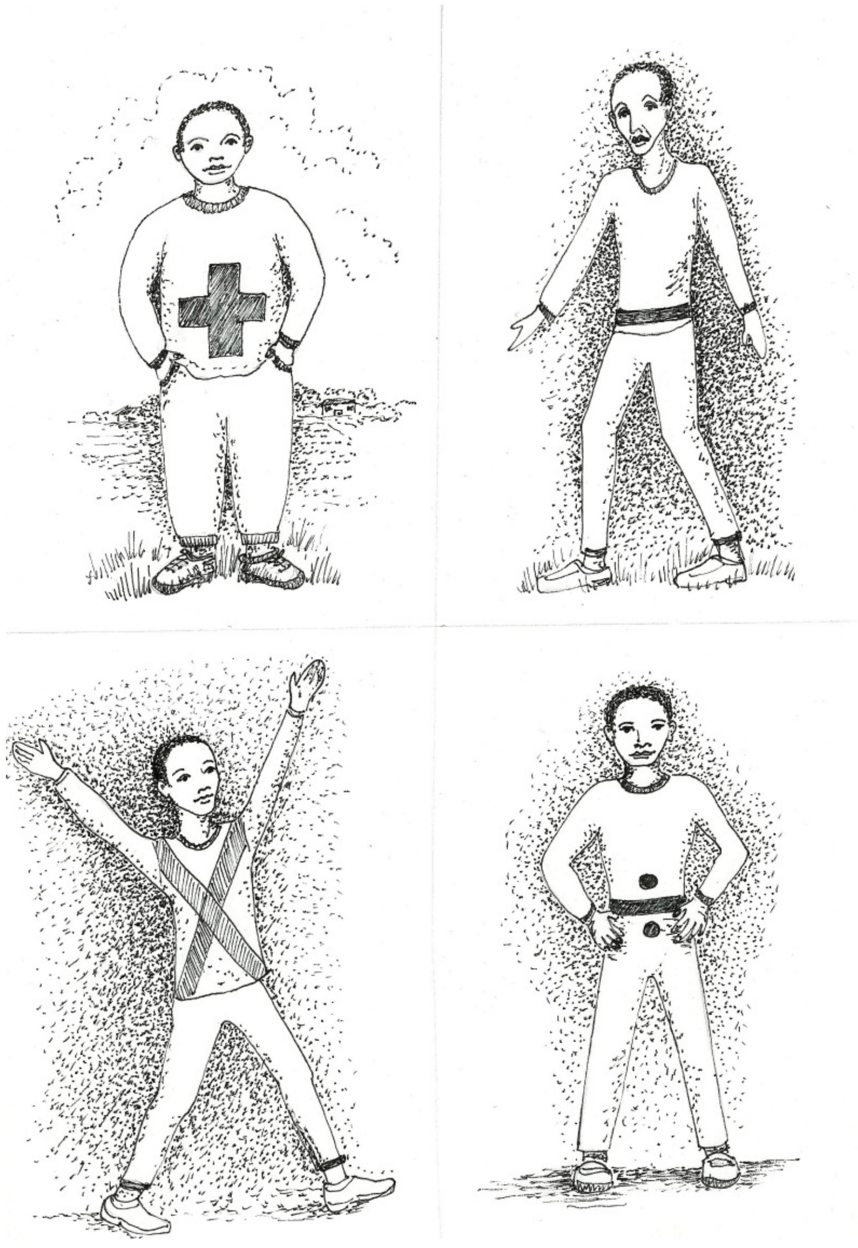
THE WALDORF APPROACH to MATHEMATICS TEACHING in GRADES 1 & 2



WRITTEN FOR

THE EAST AFRICAN WALDORF TEACHER DEVELOPMENT PROGRAM

BY

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The manuals can be downloaded from www.zukunftsstiftung-entwicklung.der

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Whilst every effort is made to acknowledge the work of others included in this manual, it has not been possible to trace the authors of the poems or other material, handed down from teacher to teacher. I ask that any information regarding authorship is passed on to me, on peterva@mweb.co.za, so that authors may be acknowledged for their work in subsequent editions/printings.

This manual is indebted to the Waldorf tradition of teaching according to the principles of Rudolf Steiner (1861 – 1925), as developed by many generations of teachers. I would like to make specific mention of the late Yvonne Bleach, my colleague from Novalis Institute and the Centre for Creative Education (both in Cape Town) and from the Federation of Waldorf Schools in South Africa, who has done so much for the development of mathematics teaching, and has been such an inspiring colleague in working together in the teaching of mathematics. Yvonne taught in the East Africa training program in August 2008, and was loved and admired by all the participating teachers.

My thanks to:
Ann Sharfman for the beautiful photograph of Yvonne Bleach on page 4.
Shamima Wolmarans for finding the two quotations printed on page 7.

Peter van Alphen

The East African Waldorf/Steiner Teacher Development Programme

The East African teacher development programme was started by the late Adeline Mlai, a Tanzanian, in Dar-es-Salaam in 1997. Adeline recognised the developmental value of Waldorf education and invited Peter van Alphen and Ann Sharfman, teacher educators with experience working in African settings in Cape Town, South Africa, to start a teacher development programme in Dar-es-Salaam. This programme was set up for teachers from Tanzania, Uganda and Kenya.

After the first year, difficulties securing the funds for continuing the programme were experienced, and in 1999 the programme was relocated to Nairobi, Kenya, as a more central venue for the three countries. The Rudolf Steiner School in Mbagathi was able to secure funding for its continuation, and in the eleven years that followed an ever-increasing number of teachers from East African countries joined the programme.

Our grateful thanks go to GLS Zukunftsstiftung Entwicklung (Bochum, Germany) and Freunde der Erziehungskunst Rudolf Steiners (Berlin, Germany) for their continued support of the programme from 1999. We also wish to thank Sanduko a Ndege (Vejle, Denmark), Internationaal Hulpfonds (Amsterdam, Netherlands), Acacia (Basel, Switserland), Stichting Helias (Netherlands) and the Iona Stichting (Amsterdam, Netherlands) for their additional support.

About this Manual

This manual answers the need for teachers (or student-teachers) to have notes on the modules they attend. This manual is written for Primary School teachers doing the module on teaching mathematics in Grades 1 and 2, which follows on the second module of the program in which a detailed study of Rudolf Steiner's concept of child development was given. The second module included details of curriculum, to show how all teaching needs to grow out of an understanding of the developmental stages of the children at each age. Teachers wishing to use this manual are therefore asked to first study the manual on Child Development, so that everything written here can be seen in the correct light.

This manual is intended to guide teachers through the difficult task of teaching Mathematics in Grades 1 and 2.

This manual is intended to be handed out at the end of the module for revision and further study. The suggestion is that participants study together in groups in their respective schools.

We trust that the material provided will be useful in Waldorf training programmes in many countries around the world. Comments and suggestions are welcomed, and can be sent to Peter van Alphen on peterva@mweb.co.za.

This manual is dedicated to the memory of

YVONNE BLEACH

Waldorf teacher and teacher-trainer, whose work in the teaching of Mathematics has enlightened and inspired countless teachers

THE WALDORF APPROACH TO MATHEMATICS GRADES 1 - 2

INDEX

The prime value of learning mathematics is that it develops our thinking, our real, willed, independent thinking; a thinking capable of directing its powers in cognising both the material world and the invisible world within.

Rudolf Steiner.

The aim of every teacher of Mathematics is to bring about a confident capacity in his or her pupils to be able to move freely within a particular topic which is being focussed upon, be it the life of number, the emergence of geometrical form and its metamorphoses, or the application of mathematical ways of thinking to practical situations and technical tasks in the world.

Ron Jarman.

LEARNING READINESS for MATHEMATICS

In Waldorf Education great emphasis is laid on children's learning readiness before they are taught to learn mathematics in a formal way. Physical development, emotional and social development, as well as intellectual development all need great care in the years before the child turns 7.

However, the primary school teacher needs to understand all these developments, as

- there may be children in her or his class that have not gone to a well developed Waldorf kindergarten, and therefore need to do catching up
- even if there is a well-developed Waldorf kindergarten, there may be children who are slow in their development, or have missed certain steps in their development, who need to do catching up
- and equally important, the primary teacher needs to build on these developments in order for the children to be able to learn mathematics well.

The foundations for Mathematics lie in the years from birth to about 7 years. This section will explain the particular developments that are needed for children to learn Mathematics, and exercises which the teacher can use as a part of every day's lessons.

PHYSICAL DEVELOPMENT

The life forces

We need to understand that the physical body consists of two parts: the physical body (our "flesh and bones") and the 'body' of 'life forces' or 'formative forces' (also called etheric forces) that brings life and development to the physical body. These life forces cannot be seen with our physical eyes, but they are present in the body, keeping the body functioning correctly. At death, these life forces leave the physical body, and so the body stops functioning and gradually decays.

In the first 7 years of a child's life, the life forces are very strong and very active. They have a great deal of work to do, because at birth the baby is not ready to live as we do on the earth: a lot of growth has to take place. The organs of the body need to be developed further to be able to function as humans do. The most important developments take place in the brain, so that the child can bring her or his body under control, can use all the senses well, and can learn to think.

During the first 7 years of life, parents and teachers need to <u>care</u> for these life forces as they need to do their work in building, developing and refining the body, so that it is fully able to meet life in our world. This means that they may not make academic demands on the child during this time, because the life forces would then be taken away from their work on the body.

Academic learning demands memory and a formal way of thinking, using up life forces that are needed to grow, develop and refine the physical body. The results of expecting academic learning before about 7 years of age are a weakening of the physical body, and later in life more easily getting serious diseases, such as nervous disorders. It can also cause ‘burn-out’, even in young children.

The child’s body gives us signs of when the life forces have completed their work. One of the main signs is the changing of teeth. The front teeth start changing before 7 years, but it is only when the first sixth tooth (counting from the middle of the teeth to the side - a molar) appears, that we can be sure that the process of developing the body has been completed.

This tells us that part of the life forces remain to keep the body growing, healthy and functioning all the time, and that part has been freed from the body, to be used for a new step in learning. Many of the other teeth will keep changing after the first sixth tooth appears, for some years afterwards.

When we consider the work of the life forces, we can see how important it is that children enter Grade One having turned 6 years of age, therefore turning 7 years during Grade One.

Movement

When a child is born, it is not ready for living on earth. Everything has to be learnt, and the child’s main way of doing this is through movement and imitating the movements of others. From moving the hands, to turning the body, to crawling and standing, the child gradually learns to walk. Speaking also is movement: the vocal cords have to vibrate, the tongue, mouth and lips all need to learn many different movements to form the different sounds.

All movements have a direct connection with the brain:

- The brain needs to develop its synapses (connections between brain cells) through learning many, many movements (eye muscles, speech muscles, large motor and fine motor co-ordination, eye-hand and eye-foot co-ordination, uprightness, crawling, walking, running, balancing, crossing the mid-line, etc)

- These synapses are the basis for thinking - without the formation of these synapses, through learning to move the body in a great number of ways, thinking is not possible

- The organs of the body need to form themselves so that the body can live in its environment

Here we see again how the body has to be prepared during this time through many different forms of movement, causing the brain to develop, before formal learning can begin. Again we can see the importance of waiting till the child turns 7 years before starting learning to write, read and do arithmetic (Maths).

EMOTIONAL DEVELOPMENT

A child needs to be a happy person to be able to learn. Self-confidence, a well-balanced emotional life and an energetic will should have been developed during the years from birth to about 7 years. Lacks in these areas may block a child's ability to learn, and the primary teacher needs to observe the emotional maturity of children carefully when they join primary school.

SOCIAL DEVELOPMENT

In a well-developed Waldorf kindergarten, plenty of opportunity is given to developing social behaviour between children, and between children and adults. As much of mathematics learning is done in whole class work, group work and working in pairs, well-developed social skills will help the teacher enormously.

INTELLECTUAL DEVELOPMENT

Once the life forces are freed, the teacher can begin to develop <u>intellectual thinking</u> in the child. This means the development of:

- the ability to <u>imagine</u> (make pictures in the mind); this can be developed by telling, rather than reading, stories to the child, imaginative teaching, drawing and painting

- the ability to <u>sequence</u> ideas (being able to put thoughts into logical order); this can be developed by recalling stories in the correct sequence, counting, challenging children by giving two or three instructions together and asking them to carry them out in the right sequence, the playing of ring games and counting games

- <u>memory</u>, which is especially developed by the learning of counting and tables, many songs and verses, recalling stories

- <u>thinking</u>, rather than merely repeating what the teacher has said; this can be stimulated by discussion between teacher and class, by teacher getting children to discuss in class (teacher making sure everyone listens to the child that is speaking); teacher presenting lesson material that has depth and gets children thinking beyond everyday life. The idea is not to ask questions to get clever, analytical answers, but rather "feeling-thinking" responses suitable in this age of childhood.

Although the life forces have been freed for learning, we have to still take great care of them. Everything the child learns needs to be presented in an imaginative way, because between the ages of 7 and 14 years the child is in the feeling, dream-like, imaginative phase and not yet ready for 'dry', abstract learning.

We have to take great care how we teach, so that we do not "rob" the child of exactly those forces that give her or him energy and aliveness. This can be done by teaching imaginatively, by means of stories, images, poems, songs and little drama exercises. In fact, the imagination stimulates the life forces and avoids children getting tired or bored.

The big danger is that teachers start teaching in a 'direct' way (the way many of us have been taught in the old, traditional style) which has a "deadening" effect on the child. It is like giving the child stones to eat, instead of the living grain which would energise and strengthen her or him.

Imagination is this living grain, because it leaves children free to picture everything in their own way. Using the imagination and the different arts (music, movement, poetry, drama, painting, drawing and modelling) is a natural way of learning for children at this age, and brings about well-being and aliveness.

MOVEMENT IN THE UNIVERSE

Movement is everywhere in us and around us. This movement is rhythmical, and we as humans constantly experience this within our bodies (heart beat, breathing, walking, etc.) and within our daily lives (day and night, the seasons, the tides of the sea, etc.). The earth, moon and planets are continually moving around the sun, which is our star. The sun is also moving as part of the galaxy that we belong to. The entire universe, however far we go, is always moving.

The Ancient Greeks spoke of the whole of creation as mathematical in its structure and movement. Everything in creation works according to harmonious and rhythmical interrelationship between numbers.

They also saw mathematics as having two faces : on the one hand Arithmetic, which has the word "rhythm" within it and has to do with numbers moving rhythmically in time, and Geometry which is numbers expressing themselves in space. "God geometrises," said Plato.

The Greeks therefore saw the whole of creation as a result of the work of numbers in both time and space, and that all life within it moves in continuous harmony and order.

We have to get the child to experience number-relationships in both time and space. Therefore arithmetic and geometry have to be given equal attention throughout schooling.

OUTER MOVEMENT TO INNER MOVEMENT

The child has mathematics in and around her or him in every way. However, this is experienced unconsciously in the body and the senses. In order to build this bodily knowledge of numbers, we do as much movement with children as possible:

- walking
- running
- crawling
- jumping (on and off things, or across things)
- climbing
- rolling
- throwing and catching
- swimming
- bean bag exercises
- games

To this, the teacher needs to add plenty of balancing exercises. One idea is to create a ‘balancing trail’ every morning before school begins, where children need to walk on low garden walls, jump from one log to the next, walk on fallen tree trunks, walk on a line drawn on the pavement, rock on balancing boards and finally walk over a balancing beam as they enter the classroom. Apart from improving their sense of balance, these exercises are especially good for building awareness of space (in other words, geometry).

Rhythmic movement
To lay a more direct foundation in the body for learning mathematics, the teacher does a lot of rhythmical movement with the children which challenges them to strongly direct their movements. They are challenged to be accurate in their movements, flowing in time to the rhythm.

In this way they consciously experience number-relationships at work using the body. If the body knows number relationships, then it is possible to build concepts of numbers (that means, to understand numbers and how they work). Examples of these activities are:

- ❖ Simple, little traditional dances (the children can sing the tune while they dance)
- ❖ Moving to songs or poems that have a strong underlying rhythm[1]:

  - ➢ short-short-long: (a dancing rhythm)

    - - — - - — - - —
    Let us dance to the rhythm of flute
    - - — - - —
    and of drum! Let us dance!

  - ➢ long-short-short: (a falling rhythm, e.g. a waterfall)

    — - - — - - — - - —
    Whispering waterfall, why do you weep?

  - ➢ short-long (a galloping rhythm)

    - — - — - — - —
    The horses gallop on the plain
    - — - — - — - —
    Away they go and back again

  - ➢ long-short (a swimming or paddling rhythm)

    — - — - — - — (-)
    Slowly swims the silent swan
    — - — - — - — (-)
    O’er the lake she rests upon
    — - — - — - — (-)
    Through the mist and then she’s gone

[1] These poems by Catherine van Alphen. Note: it is best to introduce one of these rhythms at a time, practising it till the children can do it fairly well, before introducing the next rhythm.

N.B. (-) is a silent 'short'

- ❖ Eurythmy, Bothmer, and other rhythmical movement

Learning counting and tables
Added to this, in order to learn mathematics more directly, the teacher will make up many rhythmical ways of counting and learning the tables (see pages …………), as well as movement exercises in geometric patterns (see Movement for Learning booklet for some examples).[2] These approaches develop a living connection with the world of numbers, bringing much joy and awakening much interest in children towards learning mathematics.

Keeping rhythmical movement alive and challenging
Rhythmic, purposeful movement needs to be practised on a daily basis, ideally during the main lesson time, or during one of the lessons that follow.

The teacher needs to have a sense for progression, that is, to start simply and as the class becomes good at a particular movement to make it more difficult, or to introduce another movement exercise. As soon as the children show signs of doing the movements automatically or losing interest, it is a sign that it is time to change what you are doing with them, or how you are doing it with them. Making movements more and more challenging keeps the body and senses awake and alive, stimulates the brain to learn, so that new mathematical concepts are easily taken in.

Using imagination, temperaments and bringing in plenty of variation also keep children involved and interested in what they are doing. And most important of all, for successful learning:

> *Enthusiasm and joy of the teacher makes every child want to participate!*
> *The teacher needs to create enthusiasm and joy always, every lesson!*

Remedial work
If a child cannot learn counting or working with numbers, we have to take her or him back to catching up movements with his body that she or he has missed out developing during early childhood. Crawling in particular may be needed, as sometimes children miss this step in their early development. (See Movement for Learning booklet for suggestions)

[2]The teacher can also give instructions such as "take 5 steps forward, 3 to the left, then 4 to the right". Instead of steps, these could be bunny hops, kangaroo jumps, springbuck leaps, etc

Setting high expectations
It is important to realise that movements have to be directed from the child's inner centre, and not just be a weak imitation of the teacher's movements. We therefore have to demand consciously willed movement. This requires the teacher to watch how individual children move in the group, to encourage - and expect - clear, decisive movement from each and every child (according to their ability).

THE MAIN LESSON SYSTEM

In Waldorf Schools the first part of the day is considered to be the most valuable time for children to learn new material. For this reason the first lesson is called the 'Main Lesson.' Main Lessons are arranged in blocks of three weeks to handle a certain topic (sometimes they can be two weeks, sometimes four weeks or longer, depending on the topic).

A Main Lesson block allows children to go into one topic in depth, by working on it every day. The teacher deliberately plans his or her main lessons to attain this depth, by presenting the material to be learnt in an imaginative way, and by finding creative, playful ways of practising the new material learnt.

The time of a Main Lesson is two hours, which may seem like a long time, but it is made up of three parts, each part more-or-less 40 minutes in length, appealing to a different way of learning:

- The first 40 minutes: the 'rhythmic' or 'feeling' part, in which rhythmic movement (including the learning of counting, multiples and tables), and activities which are rhythmic such as singing, speech exercises and speaking poetry, are done
- The second 40 minutes: the 'content' or 'thinking' part, in which the new material to be learnt is presented (often by means of a story or image), leading to understanding the new material
- The last 40 minutes: the 'task' or 'doing' (or 'will') part, in which children are actively working on what they have learnt; the teacher finds creative tasks for the children to consolidate the new material learnt.

After the main lesson there is a break, followed by three 40-minute lessons (another break and more lessons from Grades 3 upwards) for a variety of purposes:

- Practice lessons, to practice language and maths skills
- 2nd and 3rd language lessons
- Artistic lessons
- Craft lessons
- Games lessons

The weekly timetable for Grades 1 and 2 can look something like the following:

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
8.00 – 10.00	Main Lesson (in blocks, alternating Form Drawing, 1st Language and Maths)				
	Short Break				
10.30 – 11.10	Practice	2nd Language	Practice	2nd Language	Practice
11.10 – 11.50	3rd Language	Painting	3rd Language	Hand work	Science[3]
11.50 – 12.30	Eurythmy	Painting	Games	Handwork	Science

Main Lesson Blocks

Teaching in main lesson blocks has four distinct advantages:

- The children handle one subject in great depth for 3 or 4 weeks continuously
- They work on the main subject during the first 2 hours of the school day when they can give their highest level of concentration
- They are allowed to forget the subject for a period of time (usually about 6 weeks); the sub-conscious 'digests' it during this time
- When the subject re-appears, it arouses tremendous enthusiasm - like meeting a long-lost, favourite friend - and can be tackled with a fresh and adventurous spirit

How to arrange the main lesson blocks in one year is something that is debated by teachers in Waldorf Schools. Two main ideas seem to have become traditional. The first favours Form Drawing as a regular block, the second Nature Stories (Science).

Exactly how the blocks are arranged depends on how many school terms there are in a year, and what length each term is. The festivals that are celebrated during the school depend on which religion or religions the children come from, and will have a further influence on how the blocks are arranged. It is common practice to try to repeat the same pattern of blocks through the year, for example:

3 weeks: Form drawing (as preparation for writing).

3 weeks: Language: Learning the consonants (approx. 8 or 9 consonants)

3 weeks: Mathematics: Learning the numbers (1 - 10 or 20)

[3] Science is taught by means of nature stories, in which children become aware of the natural world

1 week: Festival week

2 weeks: Form drawing continued

4 weeks: Language: Learning the remaining consonants

3 weeks: Mathematics: Introducing Four Operations

1 week: Festival week

2 weeks: Form drawing continued

3 weeks: Language: Learning the vowels (and remaining consonants)

3 weeks: Mathematics: Four Operations continued

1 week: Festival week

2 weeks: Form drawing continued

3 weeks: Language: Making own readers

3 weeks: Mathematics: Four Operations continued

1 week: Festival week

If the second way is chosen, Nature Stories will replace the last three Form Drawing blocks, and Form Drawing becomes a regular practice lesson in the weekly timetable. Festivals do not necessarily take a whole week: often the main lesson block continues for a few days into this week.

Structure of a Main Lesson

RHYTHMICAL SECTION

The aim of the rhythmical section of the main lesson is to awaken the children, and get them properly connected to their bodies after the night's sleep, so that good learning can follow. Also, the rhythmical section aims to establish a warm, enriched mood, so that children feel happy and full of joy. The rhythmical section also unites the class, so that the children can let go of their separateness from each other and willingly participate in all the activities.

This is done by means of activities such as movement (to develop body awareness, co-ordination, concentration and awakeness[4] to rhymes, verses and poetry), dancing, singing, speech, music-making, one or two short games, bean bag or rod exercises. Wherever possible, these activities should relate to the subject matter to be taught later in the lesson.

THINKING SECTION

This section generally falls into three parts:

Recall: This is an important activity that helps children to 'digest' and consolidate the lesson from the day before. The aim is for the children to do as much of the recall as possible, the teacher only stimulating and guiding it. The recall is not merely a memory exercise - it is to re-experience yesterday's lesson, full of imagination, depth of insight and feeling.

How the teacher introduces the recall and facilitates the class discussion makes a great deal of difference: if recalling a story told yesterday, the teacher can introduce the beginning of the story, bringing the children back into the mood and qualities described the previous day. The children can then take on the further re-telling of the story, guided by the teacher only if necessary. If the story led to learning something, the teacher can get the children talking about the new learning, not just in a factual way, but bringing out the qualities or moods that were connected.

Teacher's Presentation: The teacher prepares a presentation of new material (or a continuation of material already being studied) for every day' main lesson. This presentation needs to touch both the imagination and feelings of the child in some or other way, so that it 'feeds' the soul of the child, and not only the brain.

There are different ways to 'feed' the soul:

- the information can be transformed into a story (some examples are given in this manual)
- the information can be presented in the form of an image (some examples follow)
- the teacher can present a rich description of the new material

In many cases, the presentation will be teacher-directed - it is the teacher's gift to the children on that day. There are times when a new topic (or a continuation of a new topic) can be built up together with the children; in this case there is a free flow between children and teacher in building up the new topic.

[4] See 'Movement for Learning' booklet

When the teacher has completed her or his presentation, there is time for discussion on the topic. This can be done freely between children and teacher, or between children in pairs or small groups amongst themselves (the teacher may have to set some guidelines). This allows a healthy balance between breathing in (the teacher's presentation) and breathing out (the children's responses in discussion). The discussion helps the children to connect more deeply with the topic, and to broaden their knowledge.

Consolidation: Now the teacher makes the *transition* from the presentation he/she has just made to what needs to be learnt. In the case of a maths operation, the teacher can do the operation with the children (with counters in Class 1, for example); in the case of learning letters, how the letter emerges from the character (to begin with this is done on the next day).

Here the use of the chalkboard is essential, so that the pupils learn through the *eye* as well as through the *ear*. In older classes, the writing up of key words helps the pupils to remember the important details of the topic, to see the spelling of the key words, and to give them a framework for writing in the task given by the teacher.

THE "DOING" SECTION

In this section the teacher sets a task for the children, which brings what they have learnt into their *limbs* (into the *will*). If the presentation and discussion has inspired the pupils, they will be very enthusiastic to channel their energy into the task at hand!

The task needs to fit the age of the children, and fit into the time available (which could include finishing at home for the older children). Alternatively a task that spreads over a few days could be set, such as doing lists of sums over some days, till the task is completed (this can be done, for example, in Grade 2)

WAKING AND SLEEPING IN LEARNING

Rudolf Steiner pointed to the importance of working with a three-fold rhythm in learning. When something new needs to be learnt, three activities need to follow on each other:

- The teacher's presentation of the new topic, which needs to be given full of imagination, feeling and aliveness
- The night's sleep, during which the child is 'digesting' what was learnt
- The recall of the new topic on the next day

The quality of the teacher's presentation has a deep effect on the child's sleep life: if the imagination has been touched through a story that contains a deeper, human meaning, and was told with feeling, then the child's soul has been richly fed, and the sleep during the night will be good.

During sleep, the story is ‘digested’, and its meaning unconsciously understood. The following day, when the teacher gets the children to re-experience the story in the recall, the story has great depth for them, due to the transformation that took place in the night’s sleep. For this reason, the teacher needs to start the recall by setting the mood in which the story is to be retold by the children. She or he can do this by starting to tell the beginning of the story, full of imagination and feeling, the children continuing.

This three-fold process works in the same way if the teacher presented her or his lesson material by means of an image or rich, imaginative description, rather than a story. The important thing is that the teacher brings something that will touch the deeper levels of the child, something that will make the light of the spirit rise in the soul, bringing a sense of purpose and security in life to the child.

In getting the children to re-experience the story (or image, or rich, imaginative description), the teacher should not ask questions, as this would force the children into their intellect, destroying the feelings and intuitive will of the experience. It is important to only recall, not to question or analyse. Only later in life, when the children awaken in their thinking - especially after they have turned 12 years - their critical faculties need to be brought into their thinking.

GRADE 1: The First Weeks

Following Rudolf Steiner's recommendations, the Grade 1 year begins with a Form Drawing Main Lesson block, in which the children learn to draw the basic straight lines and curves need for writing. This is followed by a block of learning letters[5], after which the first Mathematics Main Lesson block is taught.

RHYTHMIC PART OF EACH DAY'S MAIN LESSON
During the Form Drawing and Letters blocks, time is given to start learning counting during the rhythmical part of main lesson every day. Learning counting is done with rhythmical movement, as explained in pages … . To begin with, this will take about 10 minutes of the main lesson, and as children get used to rhythmical work, a longer time can be given to this.

PRACTICE LESSONS
In addition, during the practice[6] lessons that follow the main lessons, practising number recognition and working with fingers and counters prepare the way for the first Mathematics main lesson block. For details see the descriptions below.

NUMBER RECOGNITION

Children need to be able to recognise a number of items up to at least 5, without having to count them. For example, if you lay 3 oranges in front of them, they need to immediately say "there are 3 oranges".

This is important, as mathematical concepts have to be built on this base, especially when we come to numbers higher than five. For example, when we are working with counting using the fingers, the recognition that one hand has 5 fingers makes it easy to see that:

7 is 5 and 2
8 is 5 and 3
9 is 5 and 4

and
6 is 3 and 3
8 is 4 and 4
10 is 5 and 5

Number recognition comes from our bodies: we begin by knowing that we have two arms, two legs, giving us the experience of the value of 'two'. Awareness of the fingers (through finger games) brings us to go up to five, and later up to 10. The toes are also 10, but some children may not be very aware of this. So the teacher needs to play little number games with the fingers, including sometimes the toes!

[5] In cases where children need to learn in a language not their own, this main lesson block will probably be needed for learning the new language, and the learning of letters will be started in a later block
[6] In some schools these lessons are called 'extra main lessons'

First the learning is using the body (counting with the fingers), and then it can be extended to outside the body - to objects from the child's surroundings. In our classrooms we need to have baskets full of different counters (for example, 3 baskets each with a different set of counters), enough to be able to give each child about 30, if possible. These should be gathered from one's own environment as far as possible, for example:

- Seed pods
- Small stones, more-or-less the same size as far as possible
- Buttons, also more-or-less the same size as far as possible

The teacher needs to gather the different baskets of counters before the Grade 1 year begins - these become part of the equipment the teacher must have ready for starting the year. The teacher also needs to work out a good system of handing out and collecting the counters during lessons, so as not to lose time and do everything in a well-ordered way.

It is important that these counters are more-or-less of the same size, so that counting is not confused with differences in size. Also, the counters should not roll around, causing endless problems in doing sums later.

Working with counters is valuable, as the sense of touch is stimulated, providing a bodily experience of numbers. Also, counters involve keeping the children active (using the will!) and provides a concrete[7] experience of working with numbers.

HOW TO DEVELOP NUMBER RECOGNITION

An example of developing number recognition is the following game, which can be played first with fingers and then with counters:

"Show me" game (Number Recognition):

Children keep their hands behind their backs, and when the teacher says a certain number (between 1 and 5), they *quickly* (this is the exciting bit!) bring their hands forward, showing correct number of fingers:

Teacher says:
Show me ....1! (*Children show 1 finger*)
Now count "1" by touching that finger (*Children touch the one finger, saying "1*")

Show me ....2! (*Children show 2 fingers*)
Now count "2" by touching those fingers (*Children touch the two fingers, saying "2*")

.... (etc. up to 5)

Later, mix up the numbers:

[7] concrete = a physical experience, belonging to the world, and not just thinking numbers in the mind

Show me ....5!
Show me ....2!

After a while, stop asking the children to count their fingers: they must be able to just show the correct number.

When the children can do the numbers up to 5 well, we turn the game around: the teacher shows a certain number of fingers, and the children call out the number. This is more difficult, as they now have to look to see how many fingers there are (called 'visual discrimination'). When the children are doing some work, the teacher can test each individual child using the game.

The same game can be played with counters. The fun part is to see how quickly the children can place the correct number of counters in front of them.

This game can be played:
- whole class shows
- in pairs (each child with a partner): show each other and tell each other what you have
- in groups (between 3 and 5 children)

In working with the counters, we emphasise the visual[8] recognition of numbers. The child needs to be able to recognise up to 5 items at a glance.

Auditory[9] recognition is also important, and so the teacher can ask the children to tell her or him how many beats or notes they hear (up to about 5) by:

- tapping on the desk or cardboard box
- beating on a drum
- playing a few notes on an instrument
- singing a few notes

When doing this with the children, do not let them see you tapping, beating, playing or singing, because then they ill be using their eyes to work out the answers. The teacher needs to hide what she or he is doing; for example, tap under the desk, turn your back to the children, get the children to face the back of the class, get them to put their heads on the desk as if they are sleeping and are woken up by the heavenly sounds of angels calling them to the most wonderful day (they love imaginative games!) This is important, as some children are visually dominant[10], and would therefore not learn to develop good hearing.

First, the teacher will focus on recognising items up to 5. Once this is firmly established, the "show me" game can be extended to items up to 10, using the same procedure as before; for example:

8 Visual = using the eyes
9 Auditory = using the ears
10 Visually dominant = they rely on their eyes, and therefore do not develop their ears

- show me 5 and 3 … how much do we have now? (count them by touching the fingers)
- later: show me 8 … how many in one hand how many in other? (many possible answers)
- with counters: leave a little space between the first five and the counters added on; when doing 'doubles', for example 3 and 3, or 4 and 4, this would not be necessary.

Remember, the game can be played in different ways:

- whole class shows and answers
- in pairs (with partners): show each other and tell each other what you have
- in groups (between 3 and 5 children)

By first using fingers, then counters and listening to numbers of sounds, inner concepts of numbers can be formed.

COUNTING

Counting can be learnt in a great variety of ways, so that the children need never to be bored. Even those who know how to count will enjoy counting if it is done in different ways. Here follow several suggestions and examples of how one can bring variety in the way of learning.

Learning to count becomes easy if we include the very active and alive part of the child - his will. Through rhythmic movements he learns to count, doing it as if he was playing.

The child loves the rhythmic repetition of counting over and over again. Make it a part of every day's programme – it helps to give the child a sense of security and well-being.

Counting in different temperaments (or moods) : one can use the same exercise repeatedly, without losing interest, by varying the mood (temperament) e.g.

count lightly and quickly (sanguine);
count in a strong and decisive way (choleric);
count in a slow, dreamy, happy way (phlegmatic);
count in a sad way (melancholic).

Counting can also be done with an image, for example:

now we are giants taking huge, slow steps
now we are frogs taking one "plop" after the next
now we flow like the river, swinging from side to side

Whether your classroom is large or small, it is possible to do rhythmic ways of counting:

- standing behind their desks, moving their arms, clapping and stamping their feet on the spot

- for stepping exercises, design a way of moving between the desks (in their rows) so that the whole class can participate
- if possible, push the desks to one side of the classroom or into the centre of the classroom, so children can make a circle (or double circle, if necessary)
- If you really have very little space, ask half the class to do the movement, while the other half sits on the desks and helps by counting (and clapping or tapping, etc).
- If the weather is good (no rain, not too windy, not too hot), these exercises can also be done outside.

<u>SONGS & RHYMES</u>

Balancing Elephants (counting up to 5): Children sit in a circle; a length of string or wool is pulled taut across the ring; first child walks along it slowly, placing heel in front of toe and making an elephant trunk with his arms; children walk along string, turning at the end.

One little elephant balancing
Step by step along a piece of string
Thought it such a lovely stunt
He called up another little elephant !

Two little elephants balancing
Step by step along a piece of string
Thought it such a lovely stunt
They called up another little elephant !

(when there are 5 :)

Five little elephants balancing
Step by step along a piece of string
Then the string broke, what a joke ! (Clap immediately after the word “broke”)

Down fell all the little elephant folk !!! (All fall down)

Counting up to 10:
1, 2, 3, 4, 5, once I caught a fish alive
6, 7, 8, 9, 10, then I let him go again
Why did you let him go ? Because he bit my finger so
Which finger did he bite ? This little finger on the right

Counting up to 10 [to the tune of John Brown]

We love dancing together (3x)	(All dance)
1 little dancing boy (or girl)	(One child comes to front of class, or into centre of circle; class shows 1 finger)
1 little, 2 little, 3 little dancers 4 little, 5 little, 6 little dancers 7 little, 8 little, 9 little dancers 10 little dancing boys (or girls)	(One child dances while everyone claps and sings)
We love dancing together (3x)	(All dance)
2 little dancing boys (or girls)	(Second child comes to front of class, or into centre of circle; class shows 2 fingers) (Two children dance)
(… continue up to 10)	

RHYTHMIC MOVEMENT

Counting to stepping, clapping, hopping, jumping, tip-toeing, skipping, even running !

Counting in different temperaments (or moods) : you can use the same exercise repeatedly, without losing interest, by varying the mood e.g.
count lightly and quickly (sanguine);
count in a strong and decisive way (choleric);
count in a slow, dreamy, happy way (phlegmatic);
count in a sad way (melancholic).

Counting can also be done with an image e.g. :
now we are giants taking huge, slow steps;
now we are frogs taking one "plop" after the next;
now we are ducks and waddle-waddle along

1 - 10

1. Stepping: One step for each number

   1, 2, 3, 4, 5, 6, 7, 8, 9, 10

2. Touch legs, clap hands :

Touch legs	clap hands
1	2
3	4
5	6
7	8
9	10

3.

Step,	clap
1	2
3	4
5	6
7	8
9	10

4. Clapping with a partner :

Own hands	Partner's hands
1	2
3	4
5	6
7	8
9	10

5. Show number of fingers while counting:

   1, 2, 3, 4, 5, 6, 7, 8, 9, 10

6. "Show me" game (Number Recognition) :

   Children keep hands behind backs, teacher gives instructions, child must bring hands forward, showing correct number of fingers:

   Show me ....1!
   Show me ....2!

7. With counters (e.g. stones, seed pods, beads, etc)

   a) Count up to 10, taking another counter for each count.
   b) "Show me ...." game with counters.

8. Beanbags :
   Counting while passing a bean bag in a circle, in groups, in pairs, alone from hand to hand
   Throwing beanbag up and counting on each catch
   Counting passing beanbag round middle – pass to friend

9. Listening to a drum – must not be able to see hands beating the drum

1 - 20

1. Same as for counting 1 - 10
2. Same as for counting 1 - 10
3. Same as for counting 1 - 10
4. Clapping in partners:

Touch legs	Clap own hands	Clap R hands	Clap L hands
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20

5. Fingers and arms: we can see 10 fingers when we cross arms

11	1 finger + cross arms
12	2 fingers + cross arms
13	3 fingers + cross arms
....etc	....etc
19	9 fingers + cross arms
20	cross arms twice

1 - 30 / **1 - 50** / **1 - 100**

1. It's a long way from 1 up to 30, so we have to take slow steps :

   1, 2, 3, 4, 5, ... 29, 30.

2. "Story" -

   a little child was skipping down the road (very happy etc) :

   1, 2, 3, 4, 5, 6, 7, 8, 9, 10

   then met a friend, walked hand-in-hand together:

   11, 12, 13, 14, 15, 16, 17, 18, 19, 20

   then the friend had to go back home, and the child was sad to be so alone :

   21, 22, 23, 24, 25, 26, 27, 28, 29, 30

the child had to cross a river, and jump from one stepping stone to the next :

31, 32, 33, 34, 35, 36, 37, 38, 39, 40

then had to go up a little hill, decided to run up it, working hard to get to the top :

41, 42, 43, 44, 45, 46, 47, 48, 49 50

now the child was afraid of the snakes, and so tip-toed carefully along the path :

51, 52, 53, 54, 55, 56, 57, 58, 59, 60

etc.

3. Make up many clapping, stepping or movement patterns

10 - 100

1. Flash 10 fingers:

   10, 20, 30, 40, 50, 60, 70, 80, 90, 100; 10, 20, etc.

2. Clap with partners (to right and left, children stand in their rows).

Clap own hands	Clap partner's hands
10	20
30	40
....	
90	100

3. With stepping

Right foot	Left foot
10	20
30	40
....	....
90	100

4.

Step	Clap
10	20
30	40
....	
90	100

100 - 1000

1. Stepping

   100, 200, 300 1000

COUNTING BACKWARDS

Counting backwards is far more difficult than counting forwards. Therefore we first build confidence in counting forwards, and when sufficient ability is established (at least counting up to 20), we can begin - in a playful way - with counting rhymes and counting songs. These can be full of actions, and full of fun.

Each counting rhyme or counting song can be introduced with a little story, made up by the teacher. Actions are important to keep children active and concentrating. Here are some suggestions:

- Children do actions to the meaning of the words, using the fingers to show the numbers when they are said
- Call the required number of children to the front of the class, one sitting down each time there is one less; rest of the class watch and show numbers with their fingers
- Split class into groups of 5 (or 6 or 10 as needed) and let them decide in which order the children in the group sit down (on the spot) as they go through the rhyme or song
- Go through rhymes and songs with each child moving her/his counters on the desk
- Use your own, local counting rhymes and songs wherever possible!

Farmer, farmer, going to the fair
What will you buy when you get there?
5 young sheep and 4 brown cows
3 pink piglets and 2 fat fowls
The last thing I want is a pretty duck
And if I get them all, I will be in luck.

****

5 brown buns in the baker's shop
Big and brown with sugar on top
A boy came in that very day
He bought one bun and took it away
How many buns were left that day?
"4"
4 brown buns .... etc.

(Five children stand in front of the class, one child comes to baker, gives money, the baker gives the child a bun and child goes away. Change "a boy" to "a girl" in the poem when a girl comes to buy a bun.)

****

5 little ducks, swimming in a pond
Round and round and far away
Mother duck said Quack, Quack, Quack
And 4 little ducks came swimming back

4 ducklings went for a swim one day
etc.... to
No little ducks, swimming in a pond
Round and round and far away
Father duck said Quack, Quack, Quack
And 5 little ducks came swimming back.

****

5 sea horses, frisking in the bay
Playing in the wavelets, tossing in the spray
In the bursting breaker one was washed away

Four sea-horses, frisking in the bay ....etc

****

As I was walking on the shore
Six little pearly shells I saw
Washed up upon the land
And as I watched the rolling sea
Took one away and left for me
Five shells upon the sand .... etc

****

Do the following verse with 10 counters in front of each child:

Here’s Lucy Lee, who came for tea
She came to tea with me
She ate a cake and a current bun
But left the currents every one
To count them backwards just for fun
10, 9, 8, 7, 6, 5, 4, 3, 2, 1.

****

In the following rhyme, 10 Children curl up on the carpet, pretending to be little sleeping seeds. A child is asked to be a bird and take a seed away. Ask a different child each time to be a bird.

10 little seeds asleep one day
A bird flew down and took one away
How many seeds were left that day?

"9"

9 little seeds asleep one day ......

****

Finger exercises with actions :

10 little men, lying quite still
10 little men climb up the hill
10 little men curl up small
10 little men stand straight and tall
1 little man he ran away
and 9 little men came home to stay.

9 little men, lying quite still ...... etc.

****

On the farmer's apple tree
10 red apples I can see
Some for you and some for me
Pick one off for you and me

On the farmer's apple tree
9 red apples ....etc

****

Here's Lucy Lee who came to tea
She came to tea with me
She ate a cake and a current bun
But left the currents every one.
To count them backwards just for fun
10, 9, 8, 7, 6, 5, 4, 3, 2 ......1!

WORKING WITH COUNTERS

Working practically with the counters (that means no writing at all) is an excellent preparation for learning to write numbers and later, to learn the four operations (adding, subtracting, multiplying and dividing).

It is good to start simply, using little story-sums, with counters in varying numbers, first up to 5, and later up to 10.

For example, working with 4:

> Once there was a fruit-seller who had 4 mangos. A mother came past her little stall, and saw the beautiful mangos. “Ah,” she thought, “at last I have found just what I need.” And so she bought the 4 mangos to take home. When she got to her house, her 2 children came running to her. “Mama, Mama, what have you bought?” And when they saw the 4 beautiful mangos, they asked, “Can we help you make the fruit salad?” for it was a special day: the mother’s family were coming to visit.
>
> The mother shared the mangos between the 2 children. How many mangos did each child get to cut up for the fruit salad? (*Children divide the counters into two heaps, and say that each child got two*). And so, that day, they were allowed to help mother making the fruit salad.
>
> Now put the 4 mangos together again. Some weeks later, the mother bought another 4 mangos from the fruit-seller, and brought them home. Again her 2 children came running to her, calling, “Mama, Mama, what have you bought?” This time there was no special occasion, but a friend was playing with the children at their home. Mother said, “each one of you is allowed to have one mango.” How many children were there that day? So how many mangos did she give away? (*Children say “3” and move 3 counters slightly to one side*) How many did she have left over? (*Ask children to touch what was left over; they say, “1 was left over.”*)

Now children, can you find different ways of placing the 4 counters? (Some will place them in a square, others in a line, or in other patterns).

For example, at a later stage, have a little story-sum on one day to work with the number 8:

Long ago, people used to have large families. Once there was a family with 8 children (*each child places 8 counters on the desk*). There were 4 boys – can you take 4 of the children and put them slightly to one side? … Good! Now can you show me with your counters how many girls there were? … Yes, 4 girls.

Now let us put all 8 children together again. If you have 8 children in a family, some are going to be quite grown up, and others very young, not so? Well, one fine day the 2 oldest children were ready to leave their parent's home to seek their fortune in the world (*children move 2 counters slightly to one side*). Can you show your partner sitting next to you, how many children were still left in the home?

After some years, the 2 oldest came back to visit the family, so how many were there now again? … Good! 8! Everyone was so happy, they all started dancing. Each child found a partner. Can you show the partners with your counters? How many couples (or pairs) are there? … Good! 4 pairs!

This can be followed with simple instructions, asking the children to group the 8 counters in different ways. One set of groupings for addition and subtraction, for example:

8 is 5 and how many?
Put the 8 together again.

8 is 1 and how many?
Put the 8 together again

8 is 2 and 4 and how many?

Can you find other ways to group your 8 counters? (here they can be creative in finding as many ways as possible to group 8 – even 8 is 1 and 1 and 1 and ... etc)

Another set of groupings relates to multiplication and division, for example:

How can we make nice, even patterns with our 8 counters? (here the children can have:

4 groups of 2
2 groups of 4

8 groups of 1
1 group of 8
arranging them in interesting patterns

It is important not to use formal language at this stage (do not use the words ‘plus’, ‘minus’, etc or ‘addition’, multiplication’, etc.) but to keep it as close to everyday, practical life as possible.

Grouping exercises can be done:

- individually
- in pairs
- in groups of 3 to 5 children

MULTIPLES AND TABLES

One of the most important foundations in learning mathematics, is to get to know the multiples and tables. They form the basis for so much that is needed in mathematics. If children do not know their multiples and tables, they are lost - they cannot succeed in learning mathematics.

Multiples (which can also be called the “ladders”) are formed by counting in 2’s, 3’s, 4’s, etc.:

2, 4, 6, 8, 10, 12, 14, 16, 18, 20 …

3, 6, 9, 12, 15, 18, 21, 24, 27, 30 …

Tables use the multiples, stating how many times 2, 3, 4, etc. go into the multiples:

2 = **1 x** 2	3 = **1 x** 3
4 = **2 x** 2	6 = **2 x** 3
6 = **3 x** 2	9 = **3 x** 3
etc	etc

DIFFERENT WAYS OF LEARNING MULTIPLES AND TABLES

There are three ways in which the multiples and tables can be learnt in creative ways:

- Learning them with rhythmical movement
- Learning them using counters
- Learning them through drawing and writing

MOVEMENT IN LEARNING MULTIPLES AND TABLES

Children need to know the multiples and tables completely by heart. These need to become part of them, which can only be done by repeating them all the time, till they no longer have to think to find the answers.

Instead of ‘drilling’ the multiples and tables into them, we can teach these in creative ways, using <u>movement</u>. This keeps the children actively involved, awake and interested, especially if the teacher challenges the children with more and more complicated movements while saying the multiples and tables.

The art of doing the multiples and tables to movement lies in finding as many <u>interesting ways</u> to move as possible. Children love repetition, as long as it remains interesting! If they start losing concentration, or start saying the numbers mechanically (without thinking), it is time to change the movement, so that the children are challenged and interested again.

The movements must also be <u>rhythmical</u>, so that the children are carried by the rhythm and do not get tired. So the teacher introduces a rhythm to the children first, practicing it until the rhythm is well established. Only then do the children start saying the multiples (or table) they are learning, in time with the rhythm of the movements.

The same movement can be used in many ways, to keep the interest of the children alive:

- ❖ Change the speed (for example, slowly – faster – very faster – end with a steady speed)
- ❖ Change the voice (for example, whispering – soft – loud – very loud – end softly again)
- ❖ Change the temperament (for example, do it in choleric, melancholic, sanguine and phlegmatic temperaments)
- ❖ Pretend to be warriors, dancers, elephant, buck, horses, etc.
- ❖ Make the movements more difficult
- ❖ Test children’s concentration by making wrong movements - see if they can keep to the correct movements

COUNTERS IN LEARNING MULTIPLES AND TABLES

When learning the multiples, the child can place counters in their 2’s, 3’s, 4’s, etc., saying the number; for example:

ΔΔ say “2”

ΔΔ ΔΔ say “4”

ΔΔ ΔΔ ΔΔ say “6”

etc. up to 20

WRITING IN LEARNING MULTIPLES AND TABLES

Writing the multiples in their books, for example:

2 * 4 * 6 * 8 * 10 * 12 * 14 * 16 * 18 * 20

Later, when they have learnt the 2 x table, they can write them as follows:

2 = 1 x 2	12 = 6 x 2
4 = 2 x 2	14 = 7 x 2
6 = 3 x 2	16 = 8 x 2
8 = 4 x 2	18 = 9 x 2
10 = 5 x 2	20 = 10 x 2

COUNTING IN MULTIPLES :

Counting in 2's

Here are some examples (as a teacher, one will find many other examples):

step on each number, emphasising every second number:

1 **2** 3 **4** 5 **6** 7 **8** 9 **10** 19 **20**

Stand still, snap fingers on first number, clap hands on second number:

1 **2** 3 **4** 5 **6** 7 **8** 9 **10** 19 **20**

Step on first number, stand still and give a clap on second number:

1 **2** 3 **4** 5 **6** 7 **8** 9 **10** 19 **20**

Use variations of faster/slower, louder/softer, temperaments, pretending to be giants, etc :

1 **2** 3 **4** 5 **6** 7 **8** 9 **10** 19 **20**

A lot of repetition is needed - using different movements, made up by the teacher - to learn these numbers so well, that they know them thoroughly enough to move on to the next stage:
Now make in-between numbers gradually softer and softer until no-one can hear them

(1) **2** (3) **4** (5) **6** ..... (19) **20**

( ) **2** ( ) **4** ( ) **6** ..... ( ) **20**

Once they can do this well, counting in multiples of 2 can follow, with different movements, for example:

Stepping slowly 2, 4, 6, 8, 10 20

Jumping 2, 4, 6, 8, 10 20

Each child find a partner, clap own hands on first number, clap partner's hands on second number: 2, 4, 6, 8, 10 20

Counters

Each child has 20 stones (or seed pods, buttons, etc.), puts them at the top of his table in a heap, and then takes 2 stones at a time, places them in front of him saying :

2, 4, 6, 8, 10 20

The child must clearly see each two being added, making the next numbers.

Counting

Just saying the numbers, without movement or counters: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

Practical exercises, such as counting the number of legs in the class (by counting in 2's), are excellent in making the multiples real to the children.

Writing

When they have learnt, through a lot of repetition, to count in twos, we can write the numbers in our books :

2 * 4 * 6 * 8 * 10 20

Counting forwards and backwards

Once children have learnt to count the multiples forwards, we now add learning to say them backwards:

2 * 4 * 6 * 8 * 10 * 12 * 14 * 16 * 18 * 20 * 20 * 18 * 16 * 14 * 12 * 10 * 8 * 6 * 4 * 2

Also practise only the backwards part:

20 * 18 * 16 * 14 * 12 * 10 * 8 * 6 * 4 * 2

Counting forwards and backwards can be done with:

- **Movement**: for example,

	Step R foot	Step L foot
Forwards :	2	4
	6	8
	10	Clap
Backwards :	10	8
	6	4
	2	Clap
Forwards :	2	4
	6	8
	10 ....etc	

  Forwards and backwards up to 20: (same as above)

  As an additional challenge, especially for the brighter pupils, the teacher can ask the children to count far beyond 20, and backwards again; but check that every child can count forwards to 20 without hesitation.

- **Counters**: for example, Count in 2's forwards up to 20, adding another 2 stones each time:

  2, 4, 6, 8 ... etc... 20

  Also count back backwards in 2's, taking 2 stones away each time :

  20, 18, 16, 14 ... etc 2

- **Writing in books**:

  2 * 4 * 6 * 8 * 10 * 12 * 14 * 16 * 18 * 20
  20 * 18 * 16 * 14 * 12 * 10 * 8 * 6 * 4 * 2

Counting in 3's

1. Step / step / stamp

Step	step	<u>stamp</u>
1	2	**<u>3</u>**
4	5	**<u>6</u>**
7	8	**<u>9</u>**
10	11	**<u>12</u>**
.....etc, up to		
28	29	**<u>30</u>**

2.

Clap	clap	<u>clap</u>
1	2	**<u>3</u>**
4	5	**<u>6</u>**
7	8	**<u>9</u>**
10	11	**<u>12</u>**
.....etc, up to		
28	29	**<u>30</u>**

3.

Clap	clap	<u>step</u>
1	2	**<u>3</u>**
4	5	**<u>6</u>**
7	8	**<u>9</u>**
10	11	**<u>12</u>**
.....etc, up to		
28	29	**<u>30</u>**

4.

Touch head	Touch shoulders	Clap hands
1	2	**<u>3</u>**
4	5	**<u>6</u>**
7	8	**<u>9</u>**
10	11	**<u>12</u>**
.....etc, up to		
28	29	**<u>30</u>**

5. With partners:

Clap own hands	Clap own hands	Clap partner's hands
1	2	**<u>3</u>**
4	5	**<u>6</u>**
7	8	**<u>9</u>**
10	11	**<u>12</u>**
.....etc, up to		
28	29	**<u>30</u>**

Then, as for 2's above, gradually saying the first two numbers softer and softer until they cannot be heard at all :

6. Step step <u>stamp</u>
   (1) (2) 3
   (4) (5) 6
   (7) (8) 9
   ....
   (28) (29) 30

7. Clap clap <u>clap</u>
   (1) (2) 3
   etc.

8. Clap clap <u>step</u>
   ( ) ( ) 3
   etc.

9. Finally, saying only the multiples, using different movements:
   3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Multiples forwards and backwards

Again, as with the multiples of 2, once they know the multiples of 3 very well, one can learn to say them backwards. This can be done in two stages, first from 15 to 3, then from 30 to 15, and finally from 30 all the way down to 3.

Counters

As with the multiples of 2, add 3 counters at a time for counting forwards, and take away 3 counters at a time for counting backwards.

Writing in books

3 * 6 * 9 * 12 * 15 * 18 * 21 * 24 * 27 * 30

30 * 27 * 24 * 21 * 18 * 15 * 12 * 9 * 6 * 3

Remember : The more movement you do in learning counting, the fewer failures and remedial cases you will have in your class!

Order of learning multiples and tables: starting with 2's is a good place to begin. Introducing counting in 5's and 10's can be done before or after introducing counting in 3's

LEARNING THE TABLES

When the children can count freely in multiples, we can start to learn the tables :

We take as an example the 2 x table, but you can work out similar activities for all the tables.

Stepping and clapping :

step		clap	clap	clap
2	is	**1**	x	2
4	is	**2**	x	2
6	is	**3**	x	2
.........etc ....up to				
20	is	**10**	x	2

Once the children know this way of moving with the table well, create a new variation: for example, step on the number, show how many times with the fingers, and then clap to finish the table :

step		show 1 finger	clap	clap
2	=	**1**	x	2
step		show 2 fingers	clap	clap
4	=	**2**	x	2

Another popular variation: divide the class into two, the one half says "2 is", the other half responds by saying "1 x 2". They have to be awake to say only their part of the table.

Every time the children come to the point of doing the table too automatically, the teacher knows it is time to change the movements. Invent a new movement that will challenge the children to learn the table in yet another way - they will enjoy the challenge, learning the table more deeply without getting bored.

Clap in partners :

clap both partner's hands		clap own hands		clap partner's R hand		clap own hands		clap partner's L hand
2	=	**1**	x	2
4	=	**2**	x	2
etc............				

Once they do this well, vary it, for example:

stamp foot	clap own hands	clap partner's R hand	clap own hands	clap partner's L hand
2	=	1	x	2
4	=	2	x	2
etc............				

More and more complicated movements can be created to keep children's interest, while they learn the table by repetition.

Writing :

Finally let them write the table into their books using colourful crayons:

2 = **1** **x** 2

4 = **2** **x** 2

etc.

Reversing the Table :

When the children really know the table this way, reverse the order of the table and learn it the other way (it is very important for children to learn to be flexible, apart from consolidating learning the table):

1 x 2 = 2
2 x 2 = 4
3 x 2 = 6
etc.....

Note that the operative part of the table is the 1x, 2x, 3x. This needs to be emphasised.

For additional practice, emphasise the sum:

1 x 2 = **2**
2 x 2 = **4**
3 x 2 = **6**
etc.......

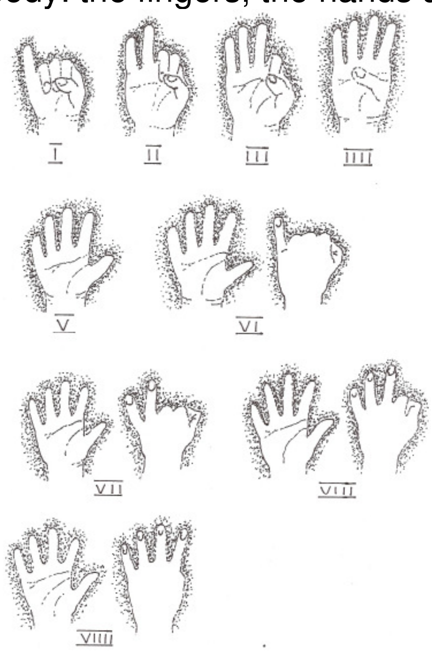
THE FIRST MATHS MAIN LESSON BLOCK

Ready for learning?
In cases where classes have mainly second language speakers, one needs to question whether the children have enough of the language used by the teacher to understand what she or he is explaining. Mathematics requires fairly complex language, and if children cannot understand, they will feel lost and unable to learn. In that case the teacher needs to postpone this main lesson block to a later time, and spend more time on learning the new language.

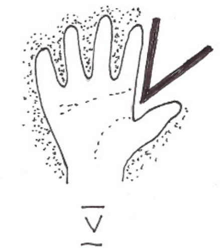
Also, before one can teach children to write numbers, they need to be able to count, at least up to about 20.

Learning the numbers up to about 10
In this first main lesson block, the children will be learning how to write numbers. However, this should not be a mechanical process, but full of meaning and inspiration. We do this by experiencing the qualities of each number, so that the writing of it is a rich and meaningful event. Because of this focus, this main lesson block is usually called the "Quality of Numbers" Main Lesson.

Why first Roman Numerals?
The Romans had a very simple but clear way of writing numbers. Although these numbers are not used much today any more - we may still see them on some watches or clocks - they have a definite advantage in that they relate to the human body: the fingers, the hands and the arms:

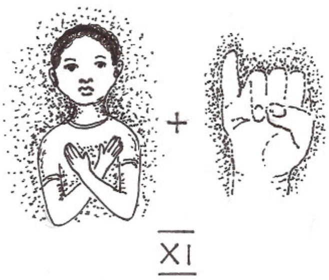


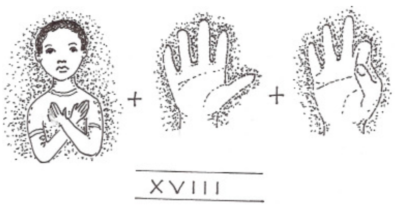
Numbers 1 to 4 are shown with the fingers of one hand
Number 5 is one hand - the V can be seen between the thumb and first finger:



Numbers 6 to 9 are shown using one hand and adding fingers from the other hand
Number 10 is shown by crossing two arms (that is, 2 hands)

To show 11, 12, etc. first cross the arms and then show the added fingers, for example:





To show 20, cross arms two times

The advantage of introducing the children to Roman Numerals before the Arabic, is that the Roman numerals show <u>a direct image</u> of the number:

Roman:	I	II	III	IIII	V	VI	VII	VIII	VIIII	X
Arabic:	1	2	3	4	5	6	7	8	9	10

Roman:	XI	XII	etc.....	XX	XXX	(L	C	M)
Arabic:	11	12		20	30	(50	100	1000)

These numbers are the way the early Romans used to write them. Later, they shortened the long numbers to save space, for example IIII came to be written as IV (1 before 5) VIIII as IX (1 before 10). However, these complications, as well as the higher numbers (L, C and M) are only taught in Grade 6, when the children study the Roman civilization.

One could tell the children that we first learn an old, old way of writing the numbers, a way that is nearly forgotten today. Of course, there will be children who know the Arabic numerals already, but they will be perfectly satisfied to explore the Roman numerals if we introduce them creatively. The Arabic numerals could then be introduced immediately after the Roman, in a pictorial way.

During the “Quality of Number” Main Lesson Block, the teacher can play enjoyable **games** as they learn more and more numbers by using the fingers, hands and arms to graphically show the numbers.

For example:

- ❖ Show me how old you are.
- ❖ Who has a younger brother or sister? Show me how old s/he is.
- ❖ Who has an older brother or sister? Show me how old s/he is.
- ❖ How old do you think your teacher is?! (If you're over 50 you'll have to find some way of showing the Roman 50 which is L ! Perhaps you are still strong and healthy, so you sit very upright on your chair! Maybe your children will enjoy 100
  years of age - by then one is very bent and old - C).
- ❖ Show me 6 in Roman. Now add 2 more - show how many - how much is that? Now take away 4 .... etc.
- ❖ Days of the week: Today is the how-many-th day of the week - don't tell me, just show in Roman numbers.

QUALITY OF NUMBERS

For children to learn the numbers, we need to introduce them in a way that fills them with wonder for the world. We need to find a way of speaking about each number, so that the 'essential nature' (or 'spiritual quality') is experienced by them. Neither the teacher, nor the children, can speak about this experience in words - it needs to be 'felt' in an intuitive[11] way.

Of course, we need words to lead us to the experience, but it is important to include the children in talking about each number. The teacher can start by talking about a number in a 'qualitative' way - a feeling-thinking way that brings out the qualities of the number. We can do this by exploring 'what is one?', 'what is two?', etc... trying to feel 'one-ness', 'two-ness' and so on up to 10 or 12. This will allow them to make a meaningful relationship with each number, so that numbers have a special place in their lives.

Here follow some suggestions you may wish to use:

The number " 1 "

MOVEMENT:

Each person is "one". Stand tall and straight - each on your own. There is only one of each of you.

Curl up: "I can't see anyone in this class! When I touch you with my magic wand, you want to uncurl and stand up straight and tall, with shining eyes. You are one person, you are one person ..." (etc, acknowledging each child as she or he stands beautifully upright.)

Now take one step, one jump, one clap (etc...)

REFLECTION / DISCUSSION:

"How did it feel to be just one?" Allow children to respond.

"So, there is only one of each of us. Of what else do we have one in the world? What is one in the world?" (Answers such as : one God, one world, one sun...... one head, one mouth, one body..........)

[11] By using the Intuition

Discussion: “How do we show one?” (One finger! One person!) “How do we write it?” (show the old (Roman) way: “I”

WORKING:

Teacher draws a number I on the board, and asks the children to “draw” it in the air, with their feet on the floor, with a finger on the partner’s back (then swap so the other child gets a turn), with the finger on the page (teacher checks each child)

Children draw the number I (allow them to draw it large) and a picture of their choice of what there is 1 in life.

The number “2”

RECALL:

Encourage the children to speak about yesterday’s experience of the number 1 and how to write it.

MOVEMENT :

Briefly repeat the experience of each person being “1”.

Take 2 steps, 2 hops, 2 jumps, 2 skips, 2 times touch knees, 2 claps ....etc

“Now get into 2’s. What does it feel like being 2?” (No talking, just feel it first.)

“What can you do when you are 2? What shapes can you make when you are 2? What happens when you pull in opposite directions, pull away from each other? How does it feel when you come together again?”

“Now just <u>be</u> 2 again (quietly together). What does it is like to be 2?”

REFLECTION / DISCUSSION:

As before. “How did it feel to be 2? Compare it to being alone.....” etc.

“Of what is there 2?” (children respond, for example, two arms, two legs, etc. ....day and night, land and see, father and mother... sickle moon and star ... good and evil.....)

WORKING:

“How do we show 2?” (2 fingers, 2 people, etc..)

Teacher draws a number II on the board, and asks the children to "draw" it in the air, with their feet on the floor, with a finger on the partner's back (then swap so the other child gets a turn), with the finger on the page (teacher checks each child)

Children draw the number II (allow them to draw it large) and a picture of their choice of what there is 2 in life.

<u>Do the same with all the numbers up to ten, or even twelve, as described.</u>

Here are some possible responses that could be drawn from the class for the other numbers:

3. Sun, moon and stars; head, trunk and limbs; father, mother and child; God the Father, God the Son and the Holy Spirit
   Movement: include making triangle with a ball of wool in groups of 3, each child holding one of the points.
   <u>Drawing</u> : include drawing of triangle (as well as a picture of their choice)

4. Four seasons; north, south, east, west; forwards, backwards, right, left
   <u>Movement</u>: include making squares with arms, later with a ball of wool as above.
   <u>Drawing</u>: include drawing of square.

5. Five -pointed star: head - 2 arms - 2 legs
   <u>Movement</u>: making a 5-pointed star with own body, with 5 children each placing one hand in the centre
   <u>Drawing</u>: five pointed star

6. 6-sided crystals; 6-sided bee-cells; Star of David (two interlacing triangles, one pointing up, one down)
   <u>Movement</u>: making a 6-sided bee-cell and Star of David in groups of 6 children holding hands,
   <u>Drawing</u>: Bee-cell and Star of David

7. 7 colours of the rainbow; 7 days of the week

8. Eight legs of a spider; eight tentacles of octopus

9. Nine months of pregnancy

10. Ten fingers, ten toes

The above 'answers' show that children use different levels of thinking:

- When speaking about 2 eyes, 5 fingers on one hand, etc, their thinking is bound up with their own bodies
- When speaking about the four legs of a table, the four wheels of a motor car, the 8 legs of a spider, etc, their thinking is bound up with external objects
- However, it is important for the teacher to stimulate 'higher thinking', that is thinking beyond the material world, using imagination, wonder, reverence, image ('picture'), ideas, qualities, etc. The teacher demonstrates through her or his way of speaking - full of feeling - how this is done. The children imitate this and learn to think in higher terms, a very valuable ability for their future.

The children could learn one new number each day. At the end of each lesson, a wonderful exercise is for the children to go into the school garden (and at home) and look for 2's, 3's, 4's, 5's, etc. in plants and trees.

The chart below shows how one could structure the daily main lessons to learn the numbers in the Quality of Numbers main lesson block:

<table>
<tr><th></th><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th></tr>
<tr><td>Rhythmic Part, about 40 – 45 minutes</td><td colspan="4">Opening of day; telling news

Movement for awakening and centering children

Movement with counting rhymes and songs
Movement with counting, counting in 2’s, repeating and varying each item to maintain interest

Songs, poems, speech exercises</td></tr>
<tr><td>Content Part, about 30 minutes</td><td><u>Introduce number I</u>:
Movement
‘Be’ 1
What does it feel like?
What is 1 in the world?

How we write I</td><td>Recall yesterday’s experiences

<u>Introduce number 2</u>:
Find partner, ‘be’ 2
Do movements
What can you do as 2?
What does it feel like?
What is 2 in the world?

How we write II</td><td>Recall yesterday’s experiences

<u>Introduce number 3</u>:
Find 2 others, ‘be’ 3
Do movements
What can you do as 3?
What does it feel like?
What is 3 in the world?

How we write III</td><td rowspan="2">Repeat same pattern in the days that follow</td></tr>
<tr><td>‘Work’ Part, about 30 - 40 minutes</td><td>Practice drawing in air, on backs, in sand, etc “draw” with finger on page

Draw number I

Free drawing from discussion of what is I

Tell on-going story</td><td>Practice drawing in air, on backs, in sand, etc “draw” with finger on page

Draw number II

Free drawing from discussion of what is II

Tell on-going story</td><td>Practice drawing in air, on backs, in sand, etc “draw” with finger on page

Draw number III

Free drawing from discussion of what is III

Tell on-going story</td></tr>
</table>

THE STORY AT THE END OF EACH DAY’S MAIN LESSON

Children at this age need to listen to a story, or part of a story, every day. In the Quality of Numbers main lesson block there is no story introducing the numbers, so an on-going story can be told (a fairy tale from local or other culture is recommended)

VERSE FOR INTRODUCING NUMBER QUALITIES[12]

[12] From Y. Bleach & S. Maher. *Putting the Heart back into Teaching*. Cape Town: Novalis Press. Page 240.

The following poem could be learnt, one verse added per day till all the verses are known, if the children understand enough English for it to be meaningful for them:

All together we are class ONE
See, the whole wide world is one
And the brightly shining sun
Sheds its light on everyone.
All alone I stand as one,
And my heart shall be a sun.

You and I, we are TWO
And many things together do.
On two feet we walk and stand
With two eyes we view the land.
Two ears to hear what wise people[13] tell
Two hands with which to do things well.

Father, mother and child are THREE
And make one happy family.
As head and heart and limbs so strong
Make one good man who'll do no
wrong.

Forwards, backwards, right and left
FOUR different ways to do our best[14]
From East and West, from South and
North,
The brother winds come blowing forth.

We make FIVE with our head
And arms and legs outspread
Now we are most like a star,
Shining brightly from afar.

We find SIX in all that is around
See, sparkling crystals in the ground
The honeycomb made by the bee,
Six-petalled flowers we also see.

The shining rainbow shows us SEVEN
As it stretches down from heaven.
And the week has seven days
Taking us on different ways.

Wriggling spiders on EIGHT legs crawl
Spinning webs on every wall
Octopus has eight legs too,
Four times as many legs as you.

NINE is a mystery hidden away[15]
A secret to be revealed[16] one day.

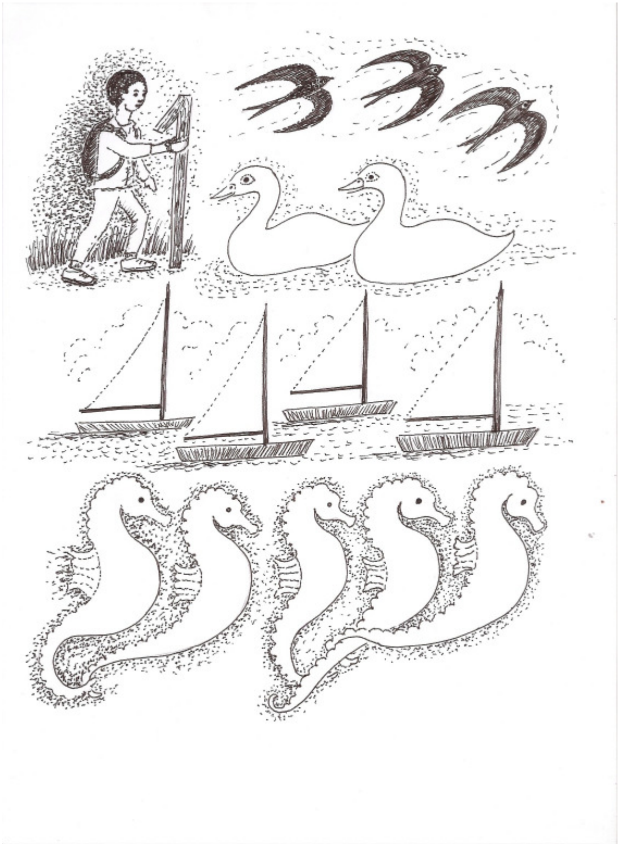
TEN fingers we have on two hands,
Ten toes, as on two feet we stand.
For ten contains within two fives
Which we'll remember all our lives.

Anonymous[17]

[13] The original poem uses the word 'men' instead of 'people'
[14] The original has 'Summer, Autumn, Winter, Spring / FOUR different seasons do us bring.'
[15] The baby in the womb is the mystery! Who knows what it will look like when it is born, or what kind of person it will grow into!
[16] Revealed = made known
[17] Author is not known

Introducing Arabic Numbers

Using a different approach now, one can learn the Arabic numbers by means of a poem, with pictures that express each number. The teacher creates a story arising out of a poem, and each day tells a part of the story, introducing one number per day:



1 Straight Staff on the road to take
2 Egyptian Geese swimming in the lake
3 Black Swifts swooping through the sky
4 Sailing Boats[18] sailing slowly by
5 Sea Horses floating on a wave
6 Sea Shells in a sandy cave
7 Flapping Flags flying in the breeze
8 Wood Owls sitting in the trees
9 Pink Petunias nodding in the sun
10, We've travelled round to where we first begun.

Catherine van Alphen

[18] Or Arab Dhows

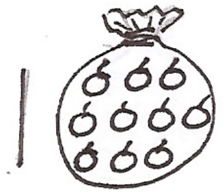
NUMBER TEN AND BEYOND

The above poem introduces the number 10 in a pictorial way: the hero of the story finds his (or her) home again. However, this picture needs explanation, so that children understand why we write 10 in this way.

This brings the children to their first understanding of ‘place value’. The ‘1’ in the number 10 stands for one ten, just like when we crossed our arms for the Roman 10, and the ‘0’ stands for ‘none’, just like in the Roman counting, when we did not add any fingers to the 10 (we could show a fist for ‘no fingers’).

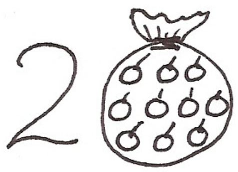
Another way is to work with the following image[19]:

> If we have a bag of 10 carrots, we can draw this bag with 10 carrots in it. Now we can write the number ‘10’ below it (*write the 1 in, say, yellow and the 0 in, say, blue; the bag itself also in yellow, the carrots in blue*):



10

> If we have 2 bags with 10 carrots in each, how many carrots do we have? (*Teacher draws two bags with 10 carrots in each, same colours as above; the children count the number of carrots, and say ‘20’*). Now look how we write ‘20’ (*write the 2 in yellow, the 0 in blue)*. This says ‘2 tens’:



20

Do the same with thirty (‘3 tens’).

[19] Thanks to Augustus Mutua, teacher at Rudolf Steiner School, Mbagathi, Nairobi for this way of explaining place value

Let the children draw all the above in their books.

We can revise the above on the next day and now go on to adding single carrots to 1 bag of ten. Write the number below each drawing in the correct colours.

Ask the children what the ‘1’ in the tens place says, and what the ‘1’, ‘2’, ‘3’, etc) in the units place says.

Writing the numbers up to 20 will complete the picture. The achievement of knowing how to write numbers up to 20 needs to be celebrated in some or other way: there needs to be great joy about this! Perhaps the children can write it beautifully on a piece of paper, and take it home to show the parents.

In the days that follow, one will have to go over the numbers 10 to 20, to make sure every child remembers how the place value works.

PRACTISING THE WRITTEN NUMBERS

There are a variety of ways to consolidate the learning of the Arabic numbers, towards the end of the main lesson block, and in the practice lessons after the block has ended. Here are some suggestions:

1. Flashcards: create your own set of flashcards, with numbers 1 – 20. For this you need 20 cards (stiff paper that will not bend easily) of the same size. On each card, draw one of the numbers - they must be drawn LARGE and THICKLY so that the children at the back of the class can read them easily. The teacher can show any of the numbers learnt already, the class (or boys/girls or one row at a time, etc) saying the number. While the class is working, the teacher can go round from child to child and test each of them using the flashcards.
2. Board work: teacher writes a number on the board, and children first whisper to each other which number it is, and then shout out the number as a whole class (nice contrast!) or show the number in the Roman way, using fingers, hands and arms as needed.
3. Teacher asks the children how they would draw a certain number, and then the children ‘draw’ it in the air in front of them, or on the desks with their finger, or even with the foot on the ground. Teacher can walk around and watch what individual children do (but the whole class is carrying out the ‘drawing’ of the numbers - keep everyone busy!)

BONDS

Bonds have to do with knowing the parts that make up a certain number. For example, the bonds of 6 are:

6 = 5 + 1	1 = 6 – 5
6 = 4 + 2	2 = 6 – 4
6 = 3 + 3	3 = 6 – 3
6 = 2 + 4	4 = 6 – 2
6 = 1 + 5	5 = 6 - 1

This is an important skill to learn, as it speeds up the ability to do addition and subtraction.

The children are already learning the bonds in the grouping activities, using counters, from the beginning of the year (**see page** ....). But now they need to learn them consciously, in a way similar to the multiples and tables.

The counters are still very useful, and can be used a lot in learning the bonds. However, once the children understand the bonds of 6, as in the above example, they need to learn them without counters, simply as calculations.

This transition from learning with counters to learning without counters is very important, as the children need to move from working with the concrete (only able to do them with objects) to being able to work with the abstract (that is, being able to do the bonds in their heads, without needing the counters).

So, we learn the bonds first with the counters, and once known well, without the counters. It is the same with learning the bonds using the fingers. It is a skill that all children need to learn, but eventually be able to calculate without the fingers.

Some children need the counters and fingers longer than others, as they are slower in making the transition from the concrete to the abstract. The teacher will always allow these children to use their counters or fingers, until they feel they can manage without them.

In Grade 1, the children learn the bonds up to 10. Check which bonds they already know from all the grouping activities done before. They should know all the bonds up to about 5, but if not, work with these till they know them.

Practise the bonds of 6, 7, 8, 9 and 10 (see bonds of 6 above):

7 = 6 + 1
7 = 5 + 2
7 = 4 + 3
7 = 3 + 4
7 = 2 + 5
7 = 1 + 6

6 = 7 - 1
5 = 7 - 2
4 = 7 - 3
3 = 7 - 4
2 = 7 - 5
1 = 7 - 6

8 = 7 + 1
8 = 6 + 2
8 = 5 + 3
8 = 4 + 4
8 = 3 + 5
8 = 2 + 6
8 = 1 + 7

7 = 8 - 1
6 = 8 - 2
5 = 8 - 3
4 = 8 - 4
3 = 8 - 5
2 = 8 - 6
1 = 8 - 7

9 = 8 + 1
9 = 7 + 2
9 = 6 + 3
9 = 5 + 4
9 = 4 + 5
9 = 3 + 6
9 = 2 + 7
9 = 1 + 8

8 = 9 - 1
7 = 9 - 2
6 = 9 - 3
5 = 9 - 4
4 = 9 - 5
3 = 9 - 6
2 = 9 - 7
1 = 9 - 8

10 = 9 + 1
10 = 8 + 2
10 = 7 + 3
10 = 6 + 4
10 = 5 + 5
10 = 4 + 6
10 = 3 + 7
10 = 2 + 8
10 = 1 + 9

9 = 10 - 1
8 = 10 - 2
7 = 10 - 3
6 = 10 - 4
5 = 10 - 5
4 = 10 - 6
3 = 10 - 7
2 = 10 - 8
1 = 10 - 9

Once the bonds of one number are thoroughly learnt, ask spot questions (asking questions on the spot); for example, when children know the bonds of 6, ask them:

6 = 3 + ?
6 = 1 + ?
6 = ? + 4
6 = ? + 5
3 + ? = 6
1 + ? = 6
4 + 2 = ?
5 + 1 = ?
4 = 6 - ?
2 = 6 - ?
6 – 3 = ?
6 – 5 = ?

When all the bonds up to 10 have been learnt in this way, mix them altogether, asking spot questions such as:

How much is 4 + 5?
6 + how much = 10?
7 – how much = 3?
4 + 4 = ?
9 – 5 = ?
etc ....

N.B.: the use of counters, especially for those children who have not 'woken up' to maths yet, will help the learning and understanding of the bonds enormously!

HOLISTIC TEACHING OF MATHEMATICS

The Fragmentation of the Human Psyche

Our world is facing one of the deepest crises of all times. It seems to be falling apart. Why is this happening?

For many centuries now western civilisation has been developing the human mind. However it has been developing the mind in a one-sided way, analysing everything and breaking things up to little fragments.

We have been taught to put everything into little boxes and to develop special languages for the tiniest bits of information. The emphasis has fallen on a vast quantity of information to be learnt or to be digested. Can the human being become a database?

The result of this kind of thinking has been a fragmentation, causing us to lose the ability to see how everything fits together in a meaningful whole.

Modern people have wanted, above all else, to think “cleverly.” This can only be achieved at the expense of developing an integrated whole. And so we live in a world where everything is splintered, everything is categorised and human communication becoming more and more difficult. It seems we have built a real Tower of Babel.

Exactly the same has happened in our own psyche. We have divorced the thinking part of ourselves from our feelings and from our own intuition. This has led, in our world, to the most horrendous crimes, worked out by very clever people, but without any connection to human feelings or the divine purpose that we experience in our lives through our intuition.

The lack of morality in our world is a direct result of developing the intellect without feelings and without intuition. We are not condemning the tremendous advances made in the sciences, in communications and in modern technology. Far from it! But what we have not understood is that together with these advances, we have to develop the human psyche.

We have to make advances in integrating ourselves in order to cope with the new possibilities that modern life has presented us with. This is the challenge of the teacher, because the foundations of all later life are laid in the education of the new generation.

<u>Re-integration</u>

Rudolf Steiner pointed out that if we are to attain real peace in today's world, we have to integrate the thinking, feeling and the will parts of the human being. Integrate the individual, and you will integrate society. He saw that holistic education is the only way in which this re-integration can be achieved on a large scale.

The basis of Rudolf Steiner's education is this integration of the human being, and the emphasis of finding the wholeness in every aspect of what we teach is its most significant contribution to resolving the crisis in education that exists today.

<u>Holistic Teaching</u>

There are three basic principles that have the effect of re-integrating the person.

1. <u>Learning at three levels</u>:

Each and every thing that the child needs to learn he has to experience by **doing** it (will level), by **feeling** it (feeling level) and by **thinking** it (thinking level). The structure of the lesson therefore, has to have these three distinct elements. The order in which these three take place depends very much on what is being taught although as a general principle the thinking is often at the end of the process: the final result.

Unless you have stirred the child in his feelings and engaged him through his will in doing what he has learnt, the ability to think it clearly and well is going to be very meagre and thin. Also, only when all three parts of the child have been engaged will the child really be able to **live** what he has learnt.

2. <u>Teaching through the imagination</u>:

Children love stories. This is because stoories really touch them deeply. They live into the series of experiences that are to them as real as ordinary life itself. When a child listens to a story every part of his soul is engaged : his thinking, his feelings and, although not in a bodily form, his will (notice the intensity with which children listen, the inner movement within the child, the exuberant energy some time after a story has been told).

The teacher will therefore take the concepts that the child has to learn and transform these imaginatively. In the case of the younger children this becomes a story. In the case of the older children this gradually moves over into what we call "imaginative concepts" or "imaginative pictures" or simply "images".

So at this age we clothe the concept in a story, to make it child-centered, to make it palatable for the child. We transform the concepts to be learnt into "imaginations" :

the dry, empty linear concepts are turned into stories of beauty and human truth. The child "sees" it all in his soul and can immediately relate to it. He also feels it in his emotions and he is stirred in his will.

When a child has lived into this imaginative idea, all his actions afterwards will be moral, enthusiastic and constructive, because his feelings have been awakened.

3. Teaching from the whole to the parts:

When we are doing something new for the child to learn, we have to present the whole to the child first. For example, when children have learnt to count and we now want to introduce them to the different operations, adding, subtracting, multiplying and dividing, we need to **present these as a totality**. We will therefore present the four operations altogether first before we analyse each one and go into details.

We create a story to introduce the children to the four operations, so that they unerstand adding, subtracting, multiplying and dividing as related to each other ("doing the same thing in different ways"). The imaginative story, which contains all the related parts, **holds them all together** both in thought, in feelings and in action. It is because of this holding together that we can begin to analyse without losing the whole.

4. Inspired Teaching:

So much of modern education has taken the magic out of life. If we only bring the matter of fact, data-like information, and the mechanics of how to manipulate numbers, it will be as if we are feeding our children stones instead of bread.

Education should never be mundane. If we only teach about cars, taxis and family members, etc. - the things they actually know about anyway - we will never reach the deeper levels of the child. A child is alive within his imagination, alive in his experiences, enthusiastic for life, and needs to have these resources tapped in order to become an empowered and fulfilled human being.

This is the main reason why we do not use textbooks in Waldorf schools. Teaching using text books does not reach the vibrancy, the curiosity and drive to discover in the child.

The best way to ensure that one's lesson has something of an inspirational nature is to incorporate the "human image". We speak of children needing to have a "body image," but in the same way they must have a "human image." For example, the four operations are really like four brothers who do the same kind of activity, but in different ways. Now if I describe one as phlegmatic another as sanguine a third as melancholic and a fourth as choleric I am touching on a basic human truth. The children instinctively feel this and that brings something of depth to them.

Unconsciously they know that the four types of brothers are the four levels within themselves. In this way the story makes real human sense. That is what we refer to as containing a human image.

Every story therefore, has to meaningful. It has to be true to life in an "essential" way. So, if I transform a new concept the children have to learn into an imaginative idea or story that has meaning and human significance in it, then I am helping the children to touch their own spiritual essence at the same time as learning the basic concepts they need in life.

By connecting children with their own spiritual essence, we give them the greatest gift that education can provide : that is, the security and the trust in life that there is meaning, that there is purpose and that it is a wonderful thing to be human. An inner contentedness begins to shine out of the eyes of the children who are given inspired and meaningful stories.

THE FOUR OPERATIONS

Preparation for the Four Operations

There are a number of skills that need to be in place before one can introduce children to the four operations. They should be able to:

- count with ease up to at least 20, ideally up to 50 or even 100
- count backwards from at least 10
- count in 2's and 3's (multiples of 2's and 3's)
- do grouping of counters with ease (see below)

Activities with Counters

The four operations are prepared long before they are taught, by means of little activities with counters. Very informally we can ask children to put five counters on the table and see how they can be arranged into groups in different ways. One can then ask children how they have grouped the 5 counters, being careful not to use the formal language of 'equals,' 'plus,' 'minus,' 'multiply,' 'divide,' etc. It is better at this stage to use the words they know, such as "5 is 1 and 4" instead of "5 equals 1 plus 4".

In doing these activities with counters we are not yet teaching the four operations, but we are allowing the children to explore the possibilities. Important is the use of varied language, for example :

> 'can you make groups of 3?'
> 'can you make up 10 in different ways?
> 'can you share out the counters on your table?' as well as 'divide them between ... friends'
> 'give away some of your counters and tell your partner how many are left'
> '5 and 2', as well as 'count 5 and then add another 2'
> '6 is 4 and how many more?'
> 'how many times can you make groups of 3 in the 9 counters?'

As the children gain confidence through a lot of this kind of activity, more advanced ways of working with the counters can be undertaken, introducing the image of the "magic circle," which always refers to the total number, i.e. the whole from which the parts are made up. Whenever they refer to the "magic circle," each child uses both arms in a gesture to show the whole, encircling the total from which one begins. An example follows:

Children work in partners (good social exercise!). Six counters are given to each group of 2 children.

The children make a “magic circle” with their hands around the six counters, saying “6 is ….” and then they touch each counter in turn, saying “1 and 1 and 1 and 1 and ……………….etc.”

The child on the right (later left, etc) puts his/her hand on some of the counters and draws them to his side of the table. Now count each group. Make magic circle round 6 counters saying “6 is ……” and then show each group saying, e.g. “4 and 2”. Compare everyone’s “sums” in the class, e.g. “6 is 5 and 1”, “6 is 3 and 3”, each group of children making the magic circle round their counters.

Put all 6 counters together. Partner on the left (later right): give your friend a present. What do you have left? (e.g. 2). Then let us say together: “2 (touch) is 6 (do magic circle) give away 4 (touch)”

Introducing the four operations

In Waldorf schools all the important new content children learn is done through main lesson blocks of three, sometimes four, weeks. The class has experienced the Quality of Numbers block (learning to write numbers while understanding the qualities of each number) some time ago now, after which other main lesson blocks (language and form drawing) have been done. A lot of preparation for learning the four operations has taken place during this time by learning to count and say the multiples through movement (especially during the rhythmical part of the main lessons), and doing work with the counters (during practice lessons).

The next Maths main lesson block introduces the four operations - addition, subtraction, multiplication and division - together as a whole. They are introduced as four characters that belong together, but are different in their own ways. They can be presented as four brothers, or four sisters, or four helpers of the king, showing the inter-relationship of the four operations (doing the same kind of things in different ways).

The stories that the teacher makes about each character need to follow the principles of using the ‘human image’ and of being moral stories that encourage caring and sharing rather than greed. Each character can be modelled on one of the four temperaments, so that the different types of children in the class are all acknowledged:

Addition	-	appeals to phlegmatics, who like to count slowly and methodically, who gain pleasure out of counting and storing.
Subtraction	-	appeals to melancholics, who know suffering and will always care for others who are facing difficulties; they love to give things away, making others happy again
Multiplication	-	appeals to sanguines, who enjoy counting quickly and lightly, enjoying big numbers
Division	-	appeals to cholerics, who like to take control and establish order into situations, sharing things out equally (strong sense of justice!), and tackle their tasks with enormous energy; they like large and difficult tasks.

The four characters can be introduced in a general way, not yet introducing each one's sign (+, -, x, ÷), to the children on the first day of the main lesson block. They could be introduced to how they are related (for example four brothers, sisters or children of the same family or four helpers of the king), what their names are and described according to what they each love doing (for example Peter Plus likes to look after the house and everything in it, Miles Minus likes to be alone and have peace and quiet so he has time to think, Timmy Times loves performing in front of others doing tricks like cartwheels, and David Divide likes organising things properly, telling others what to do). Every teacher can choose her or his own names and create a story that comes from within and will appeal to her or his class of children.

Each character can then be introduced over a three-day period. Each day a story in the life of the character is told, and out of each story comes a sum, which is first done with counters, then spoken, and finally written down.

Rudolf Steiner remarked that introducing all four operations together had two significant advantages:

It is an economical way of teaching. The children need less time to grasp the different operations, because they <u>see the inter-relationships</u> between all four operations immediately from the stories they are told.

It generates tremendous energy and aliveness in the children, because the operations are <u>linked to real life</u> (the four types of human beings, who each solve the situations they come to in their own way).

Working from the whole to the parts in the operations

The way in which we introduce the four operations has a deep effect on the attitudes of the child. In working from the whole to the parts we have to reverse the conventional way sums are taught. If we add unrelated parts together,

e.g. **2 + 3 + 1 + 4 = ?**

we are subtly reinforcing an attitude of greed in the children: "more and more and more and more - now look how much I have". This subtly, but very definitely, encourages attitudes of greed.

If however we say: mother has a basket with 10 apples, she gives 2 to her child, 3 to the neighbour, keeps 1 for herself and puts 4 in the cupboard, then the sum looks like this:

10 = 2 + 3 + 1 + 4

We are working from the whole (in this case 10) to the parts:

(2 + 3 + 1 + 4)

and are presenting an attitude of caring for everyone's needs rather than only taking items for oneself.

The same applies to multiplication, where the main concept to be understood is that of 'how many times'. The whole is how many one has altogether. For example, if there were 30 leaves of bread to be taken into the front of the bakery for selling to customers and Timmy Times could only carry 5 loaves of bread at a time, how many times did he have to carry the loaves?

30 = ? x 5

The totality of 30 is made up of 6 groups of 5 loaves. As in addition, we start with the whole (30) and proceed to the parts. Of course, Timmy Times is very active and loves running up and down the baker's shop many times - an idea that appeals to the sanguine children, and can be understood by all.

As in addition, multiplication needs to be taught in a moral way, of seeing the whole made of its parts, rather than multiplying a number to make up a vastly larger number (the tendency towards greed in economics).

In <u>subtraction</u>, we prefer 'giving away' which is a more moral attitude than 'taking away.' We still work from the whole, but because of the nature of subtraction, in which one did have a totality, one is now left with a certain number after having 'given away' the rest. We appeal to the melancholic whose ability to feel another's suffering will make him want to reach out and help. We can say, for example that Miles Minus had 6 jewels, but he gave many away to others in need. Although he started with a totality of 6, the result is that he still has 2 left. The subtraction sum therefore looks as follows:

$$2 = 12 - ?$$

The 2 that he has left came from a wholeness of 6 (with which he began), after he had given many away to help others (how many?).

In <u>division</u>, the main concept now is 'sharing,' with the emphasis on sharing <u>equally</u>. This appeals to cholerics, who like to see things happening according to fair principles, and love ordering life so it can work properly. Although the totality was, say, 12 cup-cakes, they needed to be shared equally between 4 children. The reality is that each child receives 3. Therefore, starting from the reality, we go to the history of 12 being shared between 4 children.

$$? = 12 \div 4$$

The sharing principle here encourages attitudes of giving a rightful part of the totality to each one equally and fairly.

<u>Creating story-sums</u>

Each teacher creates stories to introduce the four operations, specifically for the class she or he teaches. Here are some guidelines for creating your own stories:

1. A story has three parts to it:
   - Beginning: setting the scene within which the story takes place, introducing the main characters
   - Middle: here the story moves into the drama that takes place: a difficulty arises, which the hero has to overcome or solve
   - End: the difficulty is overcome and there is great rejoicing
2. Plan the story to have a new section each day for 3 days:
   - The first day, introduce the main character in a lot of detail: what does he look like; what sign does he have on his clothes; give some idea of what kind of person he or she is. Then lead the story into the middle part (the drama, the difficulty to be faced) in which a mathematical problem appears, followed by how it is solved

- The second day, tell a new drama (new difficulty) that has to be faced using the same operation, in which a new mathematical problem appears, followed by how it is solved
- The third day, tell another new drama (another new difficulty) that has to be faced, again using the same operation, followed by how it is solved
- This allows the children to experience what the main character does again and again in different situations, leading them to understand the operation very thoroughly
- Do not think they will get bored by doing the same operation three days in a row - it is your story that makes each day different, and the children love the repetition involved in solving each day's situation

3. Before creating each section of your story, first work out the sum that the story will lead to, and build your story to lead to that sum. Ask yourself which part of the sum will be the question that needs to be worked out by the children (for example, 6 = 4 + ?). The children will work out this question after the story to find the answer.

The Three-Day Lesson Plan For Introducing Each Operation

DAY		DAY	
1	Introduce 4 brothers/sisters/helpers	8	1st story of Subtraction
2	1st story of Addition	9	2nd story of Subtraction
3	2nd story of Addition	10	3rd story of Subtraction
4	3rd story of Addition	11	1st story of Division
5	1st story of Multiplication	12	2nd story of Division
6	2nd story of Multiplication	13	3rd story of Division
7	3rd story of Multiplication	14/15	Revise all operations (new stories)

	Intro Day	Day 1	Day 2	Day 3
Rhythmic Part, about 40 minutes	Opening of day; telling news Movement for awakening children; Movement for counting, learning multiples (count in 2s, 3s, 5s, 10s), maybe table of 2? Number songs and rhymes with actions Singing Speech exercises and artistic poetry			
Content Part, about 40 minutes	Tell Introductory Story of 4 brothers sisters, or helpers (where they live, their characters, what they like doing, etc)	Recall yesterday's Introductory Story Tell story of (Peter Plus), describing him Fully; the problem he has to solve Children use counters to DO the sum Teacher leads class to SAY the sum (a few times) Teacher shows class how to WRITE the sum (explain a few times using the story)	Teacher's drawing of main character is on the board Recall yesterday's story Tell new story of (Peter Plus); a new problem he has to solve Children use counters to DO the sum Teacher leads class to SAY the sum (a few times) Teacher shows class how to WRITE the sum (explain a few times using the story)	Recall yesterday's story; lead children to find essence of main character's work Tell another new story of (Peter Plus); another new problem he has to solve Children use counters to DO the sum Teacher leads class to SAY the sum (a few times) Teacher shows class how to WRITE the sum (explain a few times using the story)
Activity Part, about 40 minutes	Children draw from the story, using own their own imagination (on separate paper)	Children write the sum and draw the picture of (Peter Plus)	Children write the sum and draw new picture of (Peter Plus) in today's story	Children write the sum and draw new picture of (Peter Plus) in today's story

The DO-SAY-WRITE procedure[20]

When the teacher's story has ended, the children are asked to take out their counters, placing the right number on their desks. The do-say-write procedure helps them to understand the operation, following a three-fold approach:

- DO: The children place the total number (of melons bought, for example) on the desk; the teacher leads them into the parts (2 from one seller, how many from the other?) so the children discover the answer to the sum.

- SAY: The teacher leads them to 'saying' the sum, referring each part of the sum to the story, for example, if the sum is 5 = 2 + 3, children and teacher say together:

  "there were altogether 5 melons (*use gesture to show altogether)*, and that is (*the equal sign says 'is'*) 2 melons bought from the old fruit-seller at his home and (*instead of saying 'plus' at this stage*) another 3 melons bought from the young fruit-seller at the market."

- WRITE: The teacher shows the class, whilst saying the same as above, how to write the sum. While writing the sum on the board, the teacher needs to explain the symbols being used:
  - The equals sign (=): this is taught directly as saying "is" (or "that is" or "that is because", whichever is needed in linking with the story). The children simply learn by repetition that = means 'is'
  - Before writing the operation sign (in this example:+),the teacher asks the children:
    - "who came to help grandmother to work out how many melons she bought from each fruit seller?" (The children answer "Peter Plus")
    - "what sign does (Peter Plus) wear on his clothes?" The children can show the teacher using gestures
  - Now the teacher can explain that every time Peter Plus is needed to add things together, we use his sign (+), which says "plus"
  - The same method can be followed for the other operation signs (-, × or ÷)

Once the sum is written on the board, the children then read the sum (again as above) while the teacher points to each symbol (5 = 2 + 3, which they read as '5 is 2 plus 3').

In reading the sum from the board, it must always be connected with the story. This will make the sum meaningful to the children, and will help them understand why the sum is written in this way.

[20] My thanks to Prof Chris Breen from the University of Cape Town for this insight

Guidelines for this main lesson

- One has to work the children hard in the rhythmical part of the lesson, doing the following every day, with lots of varied movement: a few forms of counting (e.g. starting at 20 and ending at 40); doing counting rhymes/songs; some counting backwards (not too long or too difficult); counting in multiples (probably 2s, 3s, 5s and 10s at this stage); working on one table e.g. 2x table (or if well-known already, the 3x table)

- The rhythmic section also needs to include speech exercises, little poems, songs; some teachers include recorder playing (could also be done later in the day)

- Avoid confusion by keeping the story relatively simple, but rich in describing the characters and the feelings they experience in the drama they have to face. Too many characters, and unnecessary details, will only confuse the children

- To begin with, keep the numbers low, for example at first up to about 5 or 6, then up to about 10 and finally up to 12

- Be careful not to use the same numbers in a sum (e.g. 4 = 2 + 2), as the children will easily muddle up how the story relates to these numbers; it would be better to have e.g. 4 = 3 + 1

- Be careful not to use adult language when telling the story; speak at the children's level, always trying to use beautiful and expressive language

- After telling the introductory story, keep the children 'in' the story - do not ask questions about the story or 'analyse' it in any way, as this will force them to become objective about the story, therefore 'pulling them out of the story.' [Many of us have been taught this way in our own primary schooling, and it is still a habit in many conventional approaches to teaching in schools today - but do not do it: it destroys the effect of the story and spoils it for children]. Best to have a pause (usually the children need some time to absorb the story as a whole, and only gradually some start talking quietly) and lead them straight into drawing from the story (in this case the main character busy with his/her work) which keeps them in the mood and the 'living pictures' of the story they have just heard.

- Do not tell the answers - as a teacher you need to get the children to come to the answers themselves. So, when telling your story, you must not give the answer to the problem that is faced by your main character - tell it in such a way that the children want to work it out for themselves (of course, you will have to prevent those children who want to give the answer from saying anything – "keep it a secret," you can say to them, or let them whisper it in your ear when you have an opportunity to go round the class). The working out for themselves will, of course, be done with the help of counters, also those who know the answer in their heads already (though you may well give them an additional 'problem' to work out by themselves). As a teacher, one can easily see which children can find the answer by watching them work with the counters, and provide help to those who struggle (again, not giving them the answers but helping them to find the answer for themselves).

- Drawing: does the teacher draw the character on the board or not? Surely if you have told the story well, emphasizing the dress of the character, they should be able to draw from their <u>own</u> imagination. Then as a teacher you can 'read' where the child is, what is going on inside the child, rather than the child copying your drawing. This does not mean to say that the teacher does not do a drawing - best that the teacher's drawing is on the board on the <u>second</u> day when the children enter the classroom (this then re-inforces the story and the dress of the character).

SECOND DAY

- Recall: is not so much seeing what the children can remember (some children will do this very well, others not) but allowing the class to re-experience the story in an abbreviated form. It really is as if we say, "let's tell the story again," to re-live it again, the children taking turns to tell short bits of the story. The teacher can begin, instead of asking what the children remember, to start them off by telling, in summary form, the beginning of the story; e.g. "Children, you will remember that Peter Plus took a long time to wake up in the morning, because he liked to sleep such a lot. Once he got up, he was very keen to put on his favourite clothes …." and a child can take over from there to describe the clothes in detail. The important thing is to get back into the mood of the story again, especially allowing the feelings inherent in the story to be felt more deeply - as above, not by talking about the feelings, but just feeling them. Also as above, not to 'analyse' or ask questions about the story, but to allow the children to 'live' the story again in all its joys and sorrows, its challenges and breakthroughs

- The recall is also an oral exercise. The teacher needs to keep a record of who retells parts of the story, in order to ensure that every child gets a turn over several days, and to avoid the same children doing the recall all the time

- The focus of the recall is the story of the main character (who represents one of the operations), his/her special sign (the teacher's drawing should be on the board on the 2nd morning), what drama he/she encountered in the story and how it was solved: e.g. "Do you remember what you did with your counters yesterday? How many bundles had to be taken to the city? How many bundles did Timmy carry each time? And then, what did we find out - how many <u>times</u> did he have to run to the city?"

- After recall, the teacher can lead to the next part of the story i.e. the main story, begun yesterday, is now continued, in which a new mathematical problem is posed (a new drama in the life of the character!). The same procedure as yesterday is followed: story, counters (do-say-write) and writing and drawing in their books

- Note that the numbers in the story need to remain simple e.g. 6 = 3 x 2. If the children are confident, there can be progression from one day to next e.g. 10 = 5 x 2 and third day 12 = 4 x 3. But don't risk too high numbers, for the sake of the middle-range of the class

THIRD DAY

- Essence of what the character is like e.g. "likes doing things quickly and easily" and what he/she does e.g. "always wants to see how many times." Try to develop a feeling understanding for the operation rather than making a fixed conclusion. Do not tell the children your essence, but rather find a way of making it a discussion in which the essence emerges

- By the third day, when you are working with the counters, and once you have worked out the new problem posed on that day, you should be able to do some 'what if' variations e.g. if the main sum was 8 caskets shared between 4 children, and the class have worked out that each child will receive 2 caskets, you would say and then write this sum on the board. This could be followed by saying, "what if there had been 12 (later 16) caskets, how many would each child get then?" Once this was worked out with counters, if the children are still interested, you could ask, "let's go back to the 12 caskets …. what if there had been 6 children?" (With a very mature class, you might already be able to do this on the 2nd day, but probably only if they are learning in their mother tongue)

IN GENERAL

- Although the story is imaginative, it must still bring the child to developing clear mathematical understanding. Do not lead the child too far into fairy-land!

- Assessments during this block: one is continually assessing how each child is doing, remembering in this case that it is a first introduction to the four operations, and one is not expecting all children to have consolidated it - this will gradually happen in the practice lessons that follow and further main lesson blocks in which each operation will be practiced again and again in many different ways.

Practice Lessons

After the 2nd maths main lesson block, in which the children were introduced to the four operations, a lot of consolidation needs to be done. This is done in the practise lessons, in which little story-sums are told by the teacher, the children working out the answers with their counters and then writing the sums in their books.

A practice lesson can have a three-fold structure:

1. start with some rhythmic work (a song, some counting/multiples/a table)
2. follow with a story-sum (short, not full-length as in the main lesson block)
3. do-say-write: work it out with counters, say the sum and write it in book

One could take one operation per week, reminding the children of the introductory story where necessary.

3rd & 4th MATHS MAIN LESSON BLOCKS

In Grade 1 there should be four maths main lesson blocks during the year. The first main lesson block will usually be the Quality of Numbers block, in which they learn to write the numbers. In the second block they are introduced to the four operations.

The 3rd and 4th maths main lesson blocks are used to consolidate and extend the four operations. The children are reminded of the introductory stories, with their main characters and how they work out their sums.

As the class learns more of the following, higher numbers can be involved in their sums:

- multiples and tables:
  - 2, 3, 5 and 10 times tables
- counting forwards and backwards
  - up to 50, then up to 100

Grouping activities now become more conscious with the learning of bonds

- bonds of 6, 7, 8, 9 and 10 (e.g. 9 = 1 + 8; 9 = 2 + 7; 9 = 3 + 6; etc); these can be practised in the following ways:
  - with fingers
  - with counters
  - once well known, they can be written in books

The 3rd and 4th main lesson blocks will revise the four operations, using story-sums as the main activity. The story-sum remains the focus in each lesson, and still follows the DO-SAY-WRITE procedure. By now the children know how to do this procedure, and so the teacher does not have to lead it; she or he simply asks the children to carry it out.

After solving the story-sum, variations need to be introduced, as discussed before (see THIRD DAY, page 61).

In the 4th Maths block, simple sums not connected to the story-sum can be added, which the children can work on by themselves. They will enjoy this challenge. Some children will already be able to work them out without counters, others will still want to use their counters. The children can decide on this for themselves.

The programme could look something like this:

DAY		DAY	
1	Work on Addition story-sums	8	Work on Subtraction story-sums
2	Work on Addition story-sums	9	Work on Subtraction story-sums
3	Work on Addition story-sums	10	Work on Division story-sums
4	Work on Multiplication story-sums	11	Work on Division story-sums
5	Work on Multiplication story-sums	12	Work on Division story-sums
6	Work on Multiplication story-sums	13/14/15	Revise all operations
7	Work on Subtraction story-sums		

	Day 1	*Day 2*	*Day 3*	*Day 4*
Rhythmic Part, about 40 minutes	*Opening of day; telling news* *Movement for awakening children; Movement for counting, learning multiples (count in 2s, 3s, 5s, 10s), tables of 2, 3, 5, 10.* *Number songs and rhymes with actions* *Singing* *Speech exercises and artistic poetry*			
Content Part, about 40 minutes	*Teacher tells new story-sum with a new problem that has to be solved (e.g. addition)* *Children use counters to DO the sum and give their answers* *Teacher and children SAY the sum (a few times)* *Children tell teacher how to WRITE the sum (a few times using the story)* *Teacher introduces a few variations of the sum, asking children to DO-SAY-WRITE*	*Recall yesterday's story and main sum* *Teacher tells new story-sum with a new problem that has to be solved (e.g. addition)* *Children use counters to DO the sum and give their answers* *Teacher and children SAY the sum (a few times)* *Children tell teacher how to WRITE the sum (a few times using the story)* *Teacher introduces a few variations of the sum, asking*	*Recall yesterday's story and main sum* *Teacher tells new story-sum with a new problem that has to be solved (e.g. addition)* *Children use counters to DO the sum and give their answers* *Teacher and children SAY the sum (a few times)* *Children tell teacher how to WRITE the sum (a few times using the story)* *Teacher introduces a few variations of the sum, asking*	*Recall yesterday's story and main sum* *Teacher tells new story-sum with a new problem that has to be solved (e.g. multiplication)* *Children use counters to DO the sum and give their answers* *Teacher and children SAY the sum (a few times)* *Children tell teacher how to WRITE the sum (a few times using the story)* *Teacher introduces a few variations of the sum, asking*

		children to DO-SAY-WRITE	*children to DO-SAY-WRITE*	*children to DO-SAY-WRITE*
Activity Part, about 40 minutes	*Children write the sums and draw the picture relating to the story-sum*	*Children write the sums and draw the picture relating to the story-sum*	*Children write the sums and draw the picture relating to the story-sum*	*Children write the sums and draw the picture relating to the story-sum*

GRADE 2

The foundations laid in Grade 1 are built on in Grade 2. There will be those children who have learnt these foundations well, but there are also children who may need to go over the foundations again.
In Waldorf Education we do not fail children if they have not understood the work done in the previous year, but rather ask, “What are the reasons for the child not to have understood last year’s work?”

Some of the possible reasons are as follows:

- The child may be too young for the age group:
  - Does the school have a policy that only allows children to enter Grade 1 the year they turn 7?
  - Was the school’s policy carried out in this child’s case?
  - Did the parents give the correct age of the child?
- The child may be too dreamy:
  - Some children take longer than others to be fully awake in their consciousness. A dreamy child may still be ‘floating’ rather than ‘coming down to earth’. Such a child may need more time to learn. This is not a ‘bad’ thing, as the holding back of an early intellectual approach will develop special qualities for later in life: stronger idealism, imagination and abilities that are much needed in our world. The teacher can reach this child by being as imaginative as possible in the teaching of reading, writing and arithmetic. In this way the teacher can help the child make the transition from dreaminess to ‘coming down to earth’ using the child’s strongest ability: to use the imagination.
- The child has a learning disability: this may be caused by
  - Poor nutrition (the child did not eat the right kind of food for the brain to develop; often such a child is smaller in size than it should be)
  - Alcohol Foetal Syndrome (this is when the mother was a heavy alcohol drinker while carrying the baby in the womb; brain development was damaged during the pregnancy)
  - The child had a repressed childhood (was not allowed to develop normally, such as not being allowed to move, run, climb, play, participate in activities, etc., causing brain development not to follow its correct development (physical development of synapses, lack of co-ordination, poor senses)
- The child may have a disturbing home situation:

- As a teacher one needs to carefully find out what the home circumstances of a child who is not learning. This needs to be done sensitively, so as to win the confidence of the parents.

❖ The teacher may not have taught clearly, imaginatively or thoroughly enough:

- If there are children who have not understood because of a weakness in teaching, they may need to go over what they have not understood in Grade 1. Now they are older, they will probably be able to catch up more quickly with some extra help.

The reasons for not 'failing' children (that is, not repeating the same class again, provided they are the correct age for the class) are that they need to be kept as far as possible with their own age group. This is important because they will be going through the child developmental stages together. Another reason is that it damages a child's self-worth if he or she is seen to fail.
The teacher will be careful to always do revision of material learnt in the previous year before moving on to introducing new work. This will re-awaken what they have learnt in Grade 1, at the same time deepening their understanding of the material.

Multi-level teaching

Because there are children at different levels of learning in every class, teachers need to find ways in which to teach to a wide spectrum of ability.
When revising material from the previous year, it is important to remind the children of the stories that were used to introduce the material. This will help them to reconnect with those stories, and therefore remember what function was carried out by the particular character that was introduced.

In presenting new material, the use of the imagination is vital, as it helps children of different abilities to understand, whatever their abilities are. The imagination is the gateway to awaken their interest, and to introduce the new material in a pictorial way, rather than in a mechanical or abstract way.

Providing a class with work to do at different levels - easy sums that everyone can do, as well as slightly more challenging sums for those who can rise to the challenge, and then some difficult sums to keep the quick children engaged and satisfied - needs to be part of every day's teaching. Some examples will be given as we go along.

COUNTING

Counting forwards
By the end of Grade 1, the children should be able to reliably count up to 100.

Special attention may still need to be given to the numbers that lead to the next ten:

19 → 20; 29 → 30; 39 → 40; 49 → 50; etc... up to 99 → 100

Counting can now be extended up to 1,000. It is important to practise in sections, for example:

from 100 → 200; when that is well learnt, from 200 → 300; etc ...
later 100 → 300; or let us start from 150 and end at 250;
700 → 1,000, and finally the big day, when we can count from 0 – 1,000 (alternate boys and girls every hundred, for example, to avoid the class getting tired!)

Again, give special attention to numbers that lead to the next hundred:

199 → 200; 299 → 300, etc, as well as 279 → 280; 289 → 290, etc.

Counting backwards

By the end of Grade 1, the children should have learnt to count backwards from 20. In Grade 2, one can work on counting backwards from 100, probably best in stages, for example:

from 30 → 1; 40 → 1; etc ... until 100 → 1 can be reached.

BONDS

N.B.: the use of counters, especially for those children who have not 'woken up' to maths yet, will help the learning and understanding of the bonds enormously!

In Grade 1, the children would at least have learnt the bonds up to 10, and can reliably answer, for example:

8 = 7 + 1; 8 = 6 + 2; 8 = 5 + 3; 8 = 4 + 4; etc
10 = 9 + 1; 10 = 8 + 2; 10 = 7 + 3; etc

and answer spot questions such as:

How much is 4 + 5? 6 + how much = 10?

Subtraction bonds learnt in Grade 1 may need further practising, for example:

6 – 1 = 5
6 – 2 = 4
6 – 3 = 3
etc
7 – 1 = 6
7 – 2 = 5
7 – 3 = 4
etc

8 – 1 = 7
8 – 2 = 6
8 – 3 = 5
etc
9 – 1 = 8
9 – 2 = 7
9 – 3 = 6
etc

and most important of all:
10 – 9 = 1
10 – 8 = 2
10 – 7 = 3

Now in Grade 2, they need to learn bonds up to 20; for example:

11 = 10 + 1
11 = 9 + 2
11 = 8 + 3
etc

12 = 10 + 2
12 = 9 + 3
12 = 8 + 4
etc

all the way up to:
20 = 10 + 10
20 = 9 + 11
20 = 8 +12
etc

To develop flexibility of thinking, these bonds need to be done in different ways, for example:

10 + 1 = 11
9 + 2 = 11
8 + 3 = 11
etc

11 + 1 = 12
10 + 2 = 12
9 + 3 = 12
etc

and going backwards from 20:

20 = 10 + 10
20 = 11 + 9
20 = 12 + 8
etc

19 = 10 + 9
19 = 11 + 8
19 = 12 + 7
etc

Whenever learning bonds, always include spot questions, asking sometimes the whole class, sometimes the boys or girls, sometimes a particular row of children, sometimes individuals, for example:

18 = how much + 7?
6 + 5 = how much?
etc.....

Subtraction bonds can be learnt once the addition bonds are secure, in different ways as above, for example:

11 – 1 = 10
11 – 2 = 9
11 – 3 = 8
etc
12 – 1 = 11
12 – 2 = 10
12 – 3 = 9

etc
10 = 11 – 1
9 = 11 – 2
8 = 11 – 3
etc
11 = 12 – 1
10 = 12 – 2

9 = 12 – 3 etc

Halves and doubles up to 20 also need to be practised again and again:

1 + 1 = 2	half of 2 = 1
2 + 2 = 4	half of 4 = 2
3 + 3 = 6	half of 6 = 3
up to	up to
10 + 10 = 20	half of 20 = 10

and, as always, in as many different ways as possible, for example:

10 + 10 = 20	half of 20 = 10
9 + 9 = 18	half of 18 = 9
2 = 1 + 1	1 = half of 2
4 = 2 + 2	2 = half of 4

and, as before, asking spot questions to keep the children on their toes!

MULTIPLES AND TABLES

In Grade 1, the children will have learnt the multiples and tables of 2, 3, 5 and 10 times tables. These need to be kept alive, while further tables are introduced. Grade 2 is the year of learning many tables! They now have the awakeness and energy to learn a great deal more than before, and the teacher needs to use this so they feel satisfied that they are really learning!

Introducing the 4 and 6 x tables

As before in Class 1, we start by first learning the multiples of new tables.

One can introduce and practice the multiples of the 4 x table, and once these are well known, begin to learn the 4 x table itself. The multiples of the 6 x table can then follow, leading to learning the 6 x table.

The 4 and 6 times tables are related to the 2 and 3 times tables:
2, **4**, 6, **8**, 10, **12**, 14, **16**, 18, **20**
3, **6**, 9, **12**, 15, **18**, 21, **24**, 27, **30**

This means that the first half of the 4 and 6 times tables can be learnt very easily! We can do this through different rhythmical movements (stepping, clapping, hopping, skipping, etc), using images and temperaments, starting with the table they already know. For example, learning the 4 x table:
2, **4**, 6, **8**, 10, **12**, 14, **16**, 18, **20** (emphasise every second number)

(2), **4**, (6), **8**, (10), **12**, (14), **16**, (18), **20** (whisper the in-between numbers)

( ), **4**, ( ), **8**, ( ), **12**, ( ), **16**, ( ), **20** (leave out the in-between numbers, but still step or clap where they are supposed to be)

4, **8**, **12**, **16**, **20** (now say the first half of the 4 x table)

Each of these steps needs to be learnt thoroughly by repeating them over several days, before moving on to the next step.

For learning the second half of the table, it is important to keep emphasising that in a table we are <u>always adding</u> (we can show this by writing on the board, followed by practising with the counters and finally writing in their books):

4
4 + 4 = 8
8 + 4 = 12
12 + 4 = 16
16 + 4 = 20

From this, we can work out the rest of the 4 x table:

20 + 4 = 24
24 + 4 = 28
28 + 4 = 32
34 + 4 = 36
36 + 4 = 40

Now the second half of the multiples of 4 can be learnt by heart, with varying movements to make the learning easy and fun. For example:

Touch:	toes	knees	hips	shoulders	head
	4	8	12	16	20
	24	28	32	36	40

Clapping in partners:

	right hands	left hands	right hands	left hands	clap both hands
	4	8	12	16	20
	24	28	32	36	40

Practising the multiples of 4 with counters is very helpful, the children adding 4 counters each time till they reach 40.

Writing the multiples

Putting the first half and the second half of the table side-by-side, we can see a beautiful pattern:

4	2**4**
8	2**8**
1**2**	3**2**
1**6**	3**6**
2**0**	4**0**

This helps remembering the multiples of 4, and being able to continue counting above 40:
44, 48, 52, 56, 60
64, 68, 72, 76, 80

The next step can be learning to say the multiples of 4 backwards, first practising them from 20 to 4, then from 40 to 20, and finally all the numbers from 40 down to 4.

Once the children have learnt the multiples of the 4 x table, the teacher can now use varying movements for learning the table (see Grade 1 manual for learning tables through movement):

4 = **1 x** 4
8 = **2 x** 4
12 = **3 x** 4
etc ...

At a later stage, learn the table the other way, still emphasising <u>how many times</u>:
1 x 4 = 4
2 x 4 = 8
3 x 4 = 12
etc ...

Another useful exercise in learning the table, is to <u>show</u> the number of times with the fingers instead of saying how many times:

step		show 1 finger	clap	clap
4	=	1	x	4

step		show 2 fingers	clap	clap
8	=	2	x	4

etc ...

Further tables

The same procedure, working from the multiples of the 3 x table, can be used to learn the 6 x table.

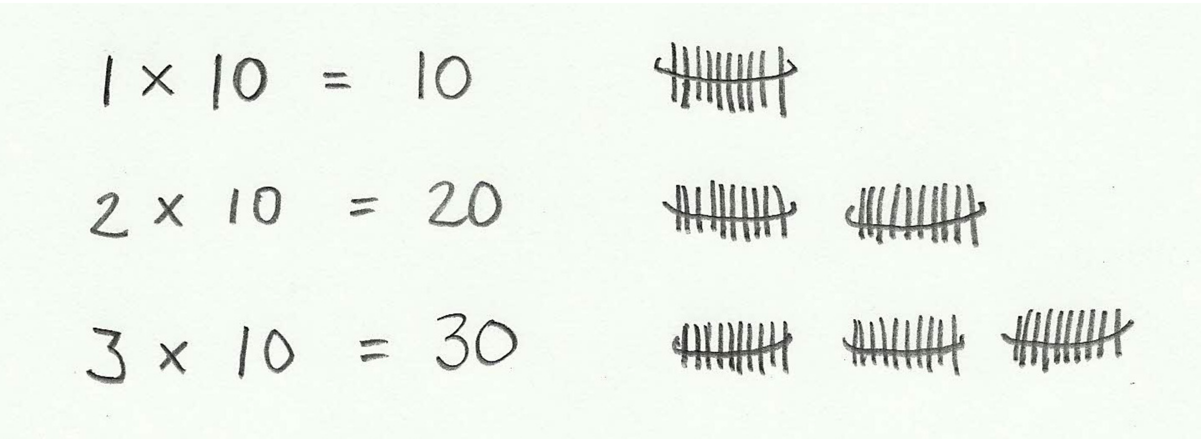
When the children are confident in learning the 4 x and 6 x tables, further tables can be added, starting first with the multiples and later learning the actual tables.

The 8 x table is related to the 4 x table, and so can be learnt in the same way as the 4 x and 6 x tables.

The teacher may, however, decide to first introduce the 9 x table, followed by the 11 x table, as these are easy to learn.

Multiplying and dividing by 10 (and later 100)

Again, starting by using match sticks (this time laying them out in rows of 10), the children can see that every time we add another row of 10 match sticks, it is another 'times 10':



1 x 10 = 10

2 x 10 = 20

3 x 10 = 30

etc .... up to

10 x 10 = 100 (said with great joy at having reached 100 so quickly!)

When it comes to dividing by 10, we have to remember what dividing means. We go back to 'sharing' between 10 people:

If I have 10 apples and share them between 10 people, how many apples will each person get?

$10 \div 10 = 1$

If I share 20 apples between 10 people, how many apples will each person get?

$20 \div 10 = 2$

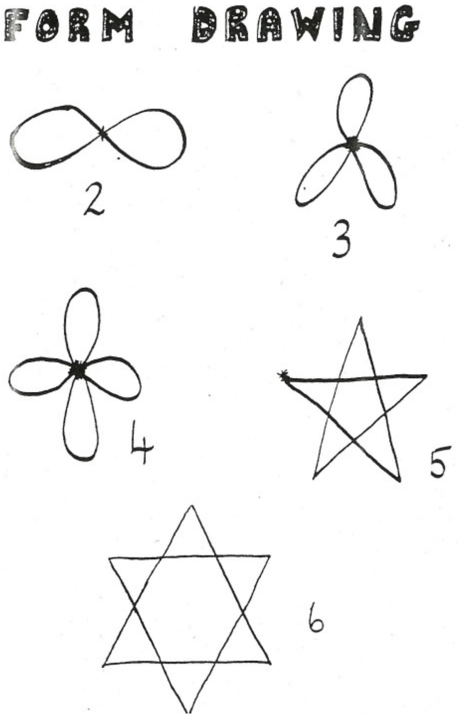
$30 \div 10 = 3$
etc ......

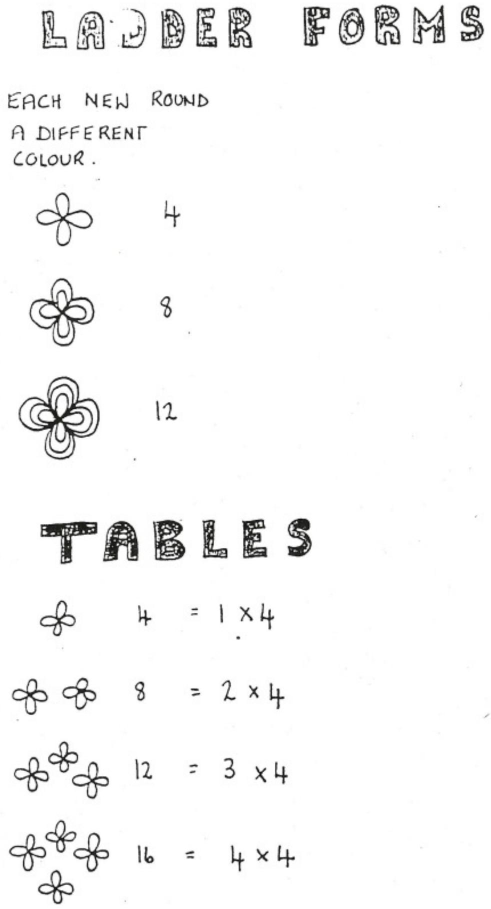
Rhythmic work

When learning multiples and tables in Grade 2, one can create more challenging movements to do. This helps children in their co-ordination, awakeness and provides fun in their learning. The creative teacher will make new patterns of clapping, stepping and other movements all the time, remembering to keep repeating the movements until the children start to do them automatically. And soon as they show signs of losing interest, of just saying the numbers without thinking, it is time to introduce a new movement pattern, learning the same numbers again and again in different ways.

Bean bags
Bean bags can be used to do more complicated movements when learning multiples and tables. The same principle of creating different movement patterns as described in 'Rhythmic work' (above) needs to be applied to learning multiples and tables using bean bags.

Number patterns (tables) (from Yvonne Bleach)





MENTAL ARITHMETIC

During maths main lesson blocks we will include practising mental arithmetic every day. When other main lesson blocks are taught, one will practise some mental arithmetic every time one has a running lesson (in some schools called ‘extra main lessons’), say for 5 minutes.

We do mental arithmetic in two ways:

Counting and simple operations

Counting, for example, from 60 to 80; from 99 to 120; in 100s or 50s or 1000s. Children of this age need to develop mental agility with numbers. So the teacher needs plenty of little number exercises e.g. Add 10 to any number. Go around the class with the first child adding 2 to the number given and the next child adding 3…[21]

Story-sums

While the children are still young (Grades 1 & 2), we do mental arithmetic mainly by means of little story-sums, told from the imagination, of life situations.

These story-sums will include:

- Bonds
- Tables
- Four operations

In story-sums on the four operations, it is important to always ask the children which operation needs to be used, for example:

> Grandmother went to the market with 12 loaves of bread. She sold 2 loaves to the farmer’s wife and another 3 loaves to the carpenter’s wife. In order to know how many loaves of bread grandmother still had to sell, which helper/brother/elder do we need to use to find this out? [Miles Minus]. Now, can you work out how many are left? [7 loaves were left over].

Those children who still need their counters should be allowed to use them, until they can let go of them.

[21] From Catherine van Alphen: Child Development Manual page 78

COUNTERS

As larger numbers will be used in Grade 2, it is best to use match sticks (cut the heads off the matches, so that there is no risk of fire), so children can bundle ten sticks together with an elastic band. This helps to understand tens and units better. Counters still play a very important role in Grade 2, for the learning the following:

- Bonds
- Place value (including 6 + 5 = 11 and 60 + 50 = 110; 3 x 4 = 12 and 3 x 40 = 120; etc...)
- Four Operations
- Problem sums
- New Concepts

THE FOUR OPERATIONS

Story-sums

The story-sum remains a most important part of each day's maths lesson. This is because the story-sum develops the ability to think clearly about a situation, to understand what needs to be done, and select the correct operation.

A story-sum interests the children because it involves human characters (sometimes animal characters!) in different life situations - the mathematical calculation then has a meaning, which they want to solve as if they are there in the situation.

If the children are learning Maths in a main lesson block, the story-sum is a longer story, and if in a practice lesson, a really short story.
In class 2 we often have two or more operations - each one following the next - in the story, so that the children develop their ability to solve 'problems' easily and speedily.

The important question the teacher keeps asking is, "which brother (or sister, helper, etc) do we need for this sum?" By reminding the children which character does which function (adding, giving away, doing something many times, sharing), they will learn more easily to select the correct operation that is needed in the sum.

'What if' variations

The story-sum can easily include some 'what-if' variations, for example, if the main sum was how many packets would the greengrocer need to pack 24 tomatoes if 4 tomatoes could fit into a packet, and the children answer '6', then a variation could be, "but what if he found some larger packets in which he could pack 8 tomatoes - how many packets would he need then?"

Just sums

As the children become more and more used to doing sums, the teacher can always add sums with no stories attached to them. Because the children's confidence has been built up in a positive way - through the stories, through a lot of work with the counters, through learning tables in a fun and lively way - they will love to do lists of sums the teacher writes up on the board.

However, there will be children at different levels of ability in the class, and the teacher therefore needs to set work at different levels (this is called 'multi-level teaching). Three columns of sums can be written on the board, each column in a different colour:

- The first column (for example written in yellow) contains sums every child can do; these are sums at the simplest level, so that even the least able children can have the satisfaction of doing a column of sums
- The second column (for example written in blue) contains sums the middle-range of ability can do; these are sums at the average level
- The third column (for example written in white) contains sums that are challenging for those who are in the highest range of ability

In each of these columns, the sums are graded from easiest to more difficult. Gradually the weakest group of children will be able to start doing the middle- range sums, and the middle-range children the more challenging sums.

Another way of arranging for children to do 'just sums' is to make cards of graded sums, graded from easiest to more and more difficult. These need to be attractively made. Even if the teacher photocopies these, he or she needs to add colour to the cards to make them child-friendly and not simply standard 'worksheets'.

Each child (or group of children sharing a card – in which case they need to be grouped according to their abilities), works from the easiest cards as far they can to other cards, according to their speed of working.

<u>Calculations in Grade 2</u>

The range of calculations in the four operations now extends to 100.

Addition and subtraction can now move towards double digits, for example:
32 + 5 = 37 (double digit + single digit, no carrying)
26 + 5 = 31 (double digit + single digit, with carrying)
39 – 7 = 32 (double digit – single digit, no carrying)
25 – 6 = 19 (double digit – single digit, with carrying)
12 + 34 = 46 (double digit + double digit, no carrying)
25 + 46 = 71 (double digit + double digit, includes carrying)
26 – 12 = 14 (double digit – double digit, no carrying)
34 – 16 = 18 (double digit – double digit, includes carrying)

Multiplication can also move towards double digits:
15 x 6 = 90 (double digit x single digit)
46 x 2 = 92 (double digit x single digit)

Division can move towards double digits, though only simple division sums (dividing by 1 digit only, no carrying, no remainders) need to be tackled at this stage:
36 ÷ 3 = 12

The method of teaching the operations is described below.

EXTENDED NOTATION

Children need to learn flexibility of thinking in maths work. This means they need to learn to do things in different ways, so that depth of understanding can be achieved. Extended notation[22] helps this process.

Though not originating from within the Waldorf tradition, there are good reasons for including aspects of extended notation.

We delay doing sums in a vertical way until place value is fully understood. This requires time, a lot of practice, and understanding of place value in each of the four operations.

Often children, who have not learnt place value thoroughly, fail to understand what one is doing in vertical sums, so it pays to lay very firm foundations first.
The basic concept of place value needs to be practised many times, to make sure children understand it thoroughly. Lists of sums can be made up, first doing them with counters, then saying them and finally writing them down. If the teacher does these full of joy, the children will be happy to learn!

[22] Notation = the way we write something; 'extended notation' means writing numbers by separating hundreds tens and units, for example: 245 = 200 + 40 + 5

11 = 10 + 1	21 = 20 + 1	31 = 30 + 1
12 = 10 + 2	22 = 20 + 2	32 = 30 + 2
13 = 10 + 3	23 = 20 + 3	33 = 30 + 3
etc.	etc.	etc.......
20 = 10 + 10	30 = 20 + 10	40 = 30 + 10

When the above are properly understood, one can do higher numbers:

56 = 50 + 6	101 = 100 + 1	110 = 100 + 10
72 = 70 + 2	102 = 100 + 2	120 = 100 + 20
96 = 90 + 6	103 = 100 + 3	130 = 100 + 30
etc.	etc.	etc. ....

NOTE: All these exercises need to be done with counters, done orally and then written in the books. Those children who do not need the counters any longer, must be left free to do the sums without them

Finally, one can introduce numbers from 110 onwards:

110 = 100 + 10	121 = 100 + 20 + 1	131 = 100 + 30 + 1
111 = 100 + 10 + 1	122 = 100 + 20 + 2	132 = 100 + 30 + 2
112 = 100 + 10 + 2	123 = 100 + 20 + 3	133 = 100 + 30 + 3
etc.	etc.	etc. ....

These are important foundation stones for understanding mathematics, and the teacher needs to check that each child has understood place value.
The next stage will be to use this approach to do the four operations.

Important to realise, is that the work outlined below needs to be done slowly, and spread through the whole Grade 2 year. Build firm foundations - do not hurry!

<u>Addition</u>

As always at this age, it is important to introduce everything that is new from story-sums that imagine real-life situations. Each story then leads to working with counters to find the answer, using extended notation, followed by saying the sum orally and writing it down as follows:

12 + 5	24 + 3
= 10 + 2 + 5 (break up 12 into 10 + 2)	= 20 + 4 + 3
= 10 + 7 (add units 2 + 5, making 7)	= 20 + 7
= 17	= 27

These may seem pedantic, but it does help understanding place value for those who need it, as well as developing a disciplined way of thinking. By means of the stories, these exercises will still be meaningful, and not simply mechanical.

> NOTE: All these exercises need to be done with counters, done orally and then written in the books. Those children who do not need the counters any longer, must be left free to do the sums without them

Once the above is understood, and well practised, adding double digits can be introduced:

15 + 12	24 + 15
= 10 + 5 + 10 + 2	= 20 + 4 + 10 + 5
= 10 + 10 + 5 + 2 (re-arrange)	= 20 + 10 + 4 + 5
= 20 + 7	= 30 + 9
= 27	= 39

Once understood, children can leave out the middle step (10 + 10 + 5 + 2, in the first example) and simply add the tens and the units. Ultimately, they should be able to do add the tens and the units in their heads, only needing to write as follows:

15 + 12 = 27 and 24 + 15 = 39

<u>Addition with carrying</u>

Addition with carrying is much easier in extended notation than in vertical notation. When the children fully understand addition without carrying, a new story-sum can introduce addition with carrying. Again, counters will make the new way of doing these sums clear to the children. It is best to use match sticks (cut the heads off the matches, so that there is no risk of fire), so children can bundle ten sticks together with an elastic band. This helps to understand tens and units better. Here are some examples, around which the teacher can create story-sums:

16 + 15	28 + 14
= 10 + 6 + 10 + 5	= 20 + 8 + 10 + 4
= 10 + 10 + 6 + 5	= 20 + 10 + 8 + 4
= 20 + 11	= 30 + 12
= 20 + 10 + 1	= 30 + 10 + 2
= 30 + 1	= 40 + 2
= 31	= 42

Of course, soon some of these steps can be left out, as children will know how these sums work. Eventually, they can shorten it to:

16 + 15 = 20 + 11 = 31 and 28 + 14 = 30 + 12 = 42

and the brightest can work it all out in their heads, simply writing

16 + 15 = 31 and 28 + 14 = 42

Later in the year, carrying with sums in the hundreds can be introduced, for example:
[first learn 5 + 6 = 11, and 50 + 60 = 110]

125 + 47
= 100 + 20 + 5 + 40 + 7
= 100 + 20 + 40 + 5 + 7
= 100 + 60 + 12
= 100 + 60 + 10 + 2
= 100 + 70 + 2
= 172

263 + 54
= 200 + 60 + 3 + 50 + 4
= 200 + 60 + 50 + 3 + 4
= 200 + 110 + 7
= 200 + 100 + 10 + 7
= 300 + 10 + 7
= 317

Subtraction

As in addition, we will keep on using story-sums to introduce a further way of working on subtraction, building on what was learnt in Grade 1. We can start very simply:

12 – 3
= 10 + 2 – 3 (can I give away 3, if I have only 2? No! But I do have 10!)
= 10 – 3 + 2
= 7 + 2
= 9

NOTE: All these exercises need to be done with counters, done orally and then written in the books. Those children who do not need the counters any longer, must be left free to do the sums without them

Similar examples can be practised many times. A step further, one could move into the 20s, 30s, etc:

24 – 6
= 20 + 4 – 6
= 10 + 10 – 6 + 4
= 10 + 4 + 4
= 10 + 8
= 18

35 – 7
= 30 + 5 – 7
= 20 + 10 – 7 + 5
= 20 + 3 + 5
= 20 + 8
= 28

When this is well known, the children are ready for double digits in subtraction.

Again, it is good to start simply:

24 – 12
= 20 + 4 – 10 – 2 (first <u>give away</u> 10, then <u>give away</u> 2; **<u>not</u>** 10 **+** 2 !!!)
= 20 – 10 + 4 – 2
= 10 + 2
= 12

Do many examples of this kind, for instance:

26 – 15
= 20 + 6 – 10 – 5
= 20 – 10 + 6 – 5
= 10 + 1
= 11

29 – 17
= 20 + 9 – 10 – 7
= 20 – 10 + 9 – 7
= 10 + 2
= 12

Once these are well known, move into 30s, 40s, etc. in a similar way.

NOTE: All these exercises need to be done with counters, done orally and then written in the books. Those children who do not need the counters any longer, must be left free to do the sums without them

<u>Subtraction with carrying</u>

Always start simply:

25 – 6
= 20 + 5 – 6 (cannot take 6 away)
= 10 + 10 + 5 – 6 (take 10 from the 20)
= 10 + 5 + 10 – 6 (re-arrange)
= 15 + 4
= 19

32 – 5
= 30 + 2 – 5 (cannot take 5 away)
= 20 + 10 + 2 – 5 (take 10 from the 30)
= 20 + 2 + 10 – 5 (re-arrange)
= 22 + 5
= 27

Gradually use larger numbers, such as:

45 – 9
= 40 + 5 – 9
= 30 + 10 + 5 – 9
= 30 + 5 + 10 – 9
= 35 + 1
= 36

64 – 7
= 60 + 4 – 7
= 50 + 10 + 4 – 7
= 50 + 4 + 10 – 7
= 54 + 3
= 57

Till we come to really big numbers (10 bundles of 10 match sticks can now be tied together with a big elastic band, making it the one hundred in the examples below), such as:

124 – 7	136 – 8
= 100 + 20 + 4 – 7	= 100 + 30 + 6 – 8
= 100 + 10 + 10 + 4 – 7	= 100 + 20 + 10 + 6 – 8
= 100 + 10 + 4 + 10 – 7	= 100 + 20 + 6 + 10 – 8
= 100 + 14 + 3	= 100 + 26 + 2
= 100 + 17	= 100 + 28
= 117	= 128

Multiplication

[It is essential that children first learn about multiplying and dividing by 10; for example: 3 x 4 = 12 and 3 x 40 = 120]

5 x 36 = ?

5 x 30 = 150 (first multiply tens)
5 x 6 = 30 (then multiply units)

[150 + 30 = 180] (add results together)

5 x 36 = 180

3 x 24 = ?

3 x 20 = 60
3 x 4 = 12

[60 + 12 = 72]

3 x 24 = 72

<u>Division</u> (could well be left for Grade 3)
60 ÷ 5 = ?
Write down multiples of 5 x table:

5
10
15
20
25
30
35
40
45
50
55

How many times 5 gives me 60?
Answer: 12 times
<u>60 ÷ 5 = 12</u>

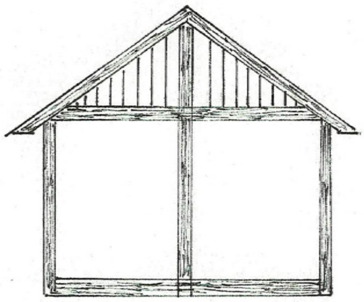
THE MATHS MAIN LESSON BLOCKS

The main lessons generally follow the following basic pattern every day. The teacher will make sure that there is a progression from each day to the next, so that the children keep learning step-by-step new skills or have to handle more difficult sums. The four operations are the main content of this year, including such topics as number patterns, extended notation and different ways of doing the four operations:

	Day 1	Day 2	Day 3
±40 mins	RHYTHMIC SECTION: Opening: Verse, Special Song, Register Movement: Multiples & Tables; Bonds; Mental Arithmetic Singing Speech exercises; poems	RHYTHMIC SECTION: Opening: Verse, Special Song, Register Movement: Multiples & Tables; Bonds; Mental Arithmetic Singing Speech exercises; poems	RHYTHMIC SECTION: Opening: Verse, Special Song, Register Movement: Multiples & Tables; Bonds; Mental Arithmetic Singing Speech exercises; poems
±25 - 30 mins	CONTENT SECTION: Story-sum; what-if variations New explanation (next step) or Revision (current step) One or two examples (next or current step)	CONTENT SECTION: Story-sum; what-if variations New explanation (next step) or Revision (current step) One or two examples (next or current step)	CONTENT SECTION: Story-sum; what-if variations New explanation (next step) or Revision (current step) One or two examples (next or current step)
±40 mins	WORKING SECTION: Sums (do the first one from the list with the class if necessary) Teacher goes round the class to help children where needed	WORKING SECTION: Sums (do the first one from the list with the class if necessary) Teacher goes round the class to help children where needed	WORKING SECTION: Sums (do the first one from the list with the class if necessary) Teacher goes round the class to help children where needed
±10 – 15 mins	STORY-TIME: Brief recall of previous story (or previous section of story) Tell new story (or new section of story)	STORY-TIME: Brief recall of previous story (or previous section of story) Tell new story (or new section of story)	STORY-TIME: Brief recall of previous story (or previous section of story) Tell new story (or new section of story)

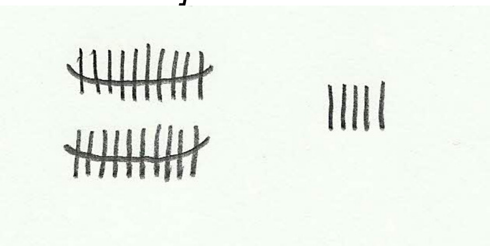
VERTICAL SUMS

When the children have understood place value thoroughly, as described above, they can be introduced to simple vertical addition and subtraction. The use of the match sticks (the teacher has to cut off the heads of the matches before giving them to the children, for safety), with the elastic bands to tie every 10 sticks into a bundle, will help the children enormously in understanding vertical addition, subtraction, multiplication and division.
One could start with a story of a woodcutter who serves his community by cutting wooden sticks and selling them for people to put up fences, make gates, build wooden huts, etc.
Every day, when the woodcutter carried his heavy load of sticks he had cut in the forest, he would swing down the huge pile on the ground behind his house. There were too many sticks to count them easily, so he never knew how many sticks he had. But one day, he had an idea! He built himself a wooden shed, in which to store all the sticks he cut every day. There was something very special about his wooden shed. He built it so that it would have two rooms. [Teacher draws the wooden shed on the board – see diagram].

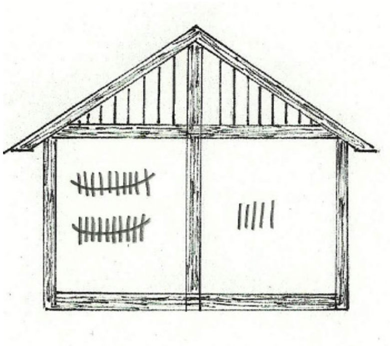


When the woodcutter had finished building his wooden shed, he was very pleased with it. "This is going to make everything so much better," he said.
At this point it would be good for each child to receive an A3 size piece of cardboard on which to draw the wooden shed. They will need a lot of guidance to divide the shed neatly into two rooms and how to draw the roof section (perhaps the teacher could draw some lines in pencil, which the children could draw over with their colour crayons). Each child can then use the 'wooden shed' she or he has made for placing the counters in the correct rooms later.
....
The next day, he went into the forest again, and cut his sticks the whole day long. Late in the afternoon, when the sun was starting to disappear behind the hills, he came walking back home, carrying the heavy load of sticks on his back. Now he went to the front of his wooden shed! There he swung down the sticks onto the ground.
Now he was very excited, because he wanted to carry out his new plan! And do you know what he did? First, he counted how many sticks he had, and found he had 25 altogether. The children now count out 25 sticks and see how many bundles
Next, he wanted to know how many bundles of 10 he could make from his 25 sticks. Every time he counted 10 sticks, he tied them with a piece of rope into a bundle! Can you do exactly what the woodcutter did, and see how many bundles of 10 he could

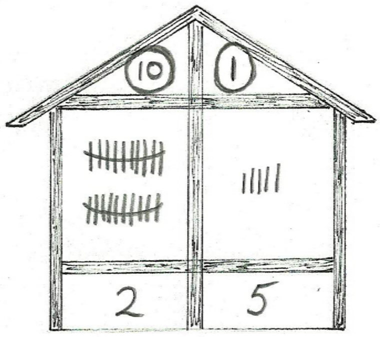
make? The children make two bundles of 10, tying each together with an elastic band, and answer *[2 bundles]. And now, how many loose sticks were left over? [5 loose sticks].*

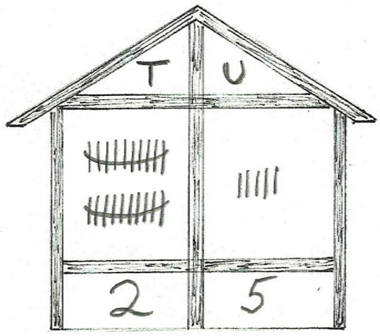


Next, he carried each bundle that was tied together, and placed them in the room on the left, which he called the 'bundles' room, and the remaining 5 sticks in the room on the right, which he called the 'loose sticks' room. [Teacher draws two bundles of 10 in the left room and 5 loose sticks in the right room; the children place their bundles and loose sticks in the correct rooms of their cardboard sheds].

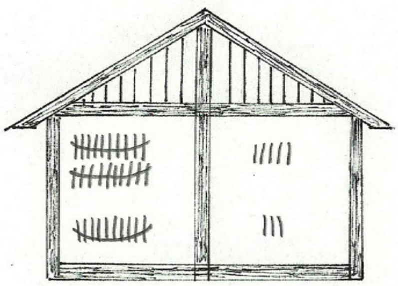


How many sticks did the woodcutter have that day? Two bundles of 10 and 5 loose sticks make how many altogether? [children answer '25'].
Let us all draw the woodcutter's wooden shed, and the number of bundles and loose sticks, in our books. Which room did the woodcutter use for the bundles of 10? [The left]. Which room for the loose sticks? [the right].
The children can draw the wooden shed with the bundles and sticks in the correct rooms in their books, copying the teacher's drawing on the board.
....
The next day, the woodcutter's story will be recalled very thoroughly, the children telling the teacher everything that happened.
A few days can be used to get the children to practise placing bundles of 10 in the left hand room and loose sticks in the right hand room.
When the teacher sees that the children are getting familiar with this way of placing the bundles and loose sticks, the idea of 'tens' and 'units' can be introduced. There are two ways of doing this, either naming them 'tens' and 'ones' (10, 1) or 'tens' and 'units' (T, U) - choose <u>one</u> of these methods, not both - and write them in the roof of the shed:



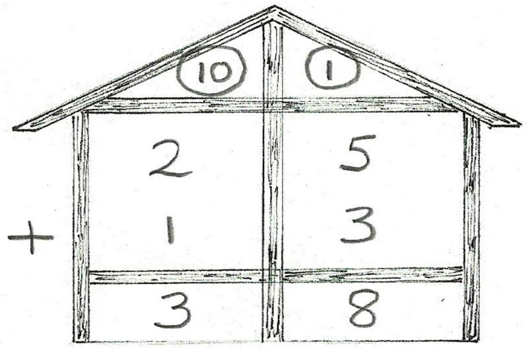


The important thing here is for the children to see that 2 bundles of 10 make 20 sticks, and that when we write the number ‘2’ under the ‘tens’ room, we mean ‘2 tens’. Get the children to always say ‘2 tens’ and never to just say ‘2’.
Practice arranging different numbers of sticks in the woodcutter’s shed for several days, <u>saying</u> how many ‘tens’ and how many ‘units’ there are each time, and <u>drawing</u> the diagrams in their books.
....
We can now introduce <u>addition</u>, doing it simply <u>without carrying</u> being involved in the sums. The story can now continue from a previous day: for example, if in yesterday’s story the woodcutter brought home 25 sticks, which were placed in bundles and loose sticks in the correct rooms, the children first place yesterday’s bundles and sticks in the correct rooms of their cardboard sheds.
The teacher now creates a new story in which a number of bundles and sticks can be added to the ones from yesterday. For example:
The woodcutter could not work as long as the previous day (the teacher creates a story as to why he was unable to work the full day), and so he brings home a smaller load of sticks. When counting his sticks, he found that he had one bundle of 10 and 3 loose sticks.
The teacher leads the children into counting out the new sticks (1 bundle of 10 and 3 loose sticks) and placing these in the correct rooms. The teacher makes the following additions to her or his drawing on the board of the wooden shed from yesterday:



How many sticks did the woodcutter carry home this day? [13]. If he adds today’s bundle of 10 and 3 loose sticks to yesterday’s sticks, how many did he have altogether now?
Allow the children as far as possible to discover how to do the addition themselves. Then go through, with the class looking at the board, how to add the bundles and loose sticks. It is important for them, from the start, to learn that you always begin by adding

the loose sticks (the units) and only then move to the bundles (the tens). When this is clear, show the children how to write the vertical addition sum, directly under the correct rooms, as follows:



The children need to say, "5 loose sticks plus 3 loose sticks is 8 loose sticks; 2 bundles of ten, plus 1 bundle of ten is 3 bundles of ten"

During the next days, you will create further addition stories of the woodcutter in which there is no carrying in the sums. These sums need to be written in their books.

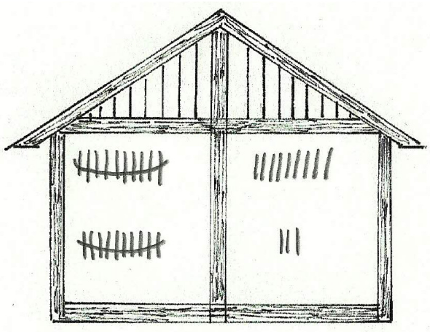
....

When vertical adding without carrying is firmly established, the teacher can move on to telling stories about the woodcutter in which the sums do require <u>carrying</u>. For example:

After a storm which broke down many big branches in the trees of the forest, the woodcutter suddenly had many fewer sticks every day, because the paths he was used to walking on were blocked by fallen branches. And so, one day he had only 19 sticks in his wooden shed. How many bundles of 10 did he have? How many loose sticks?

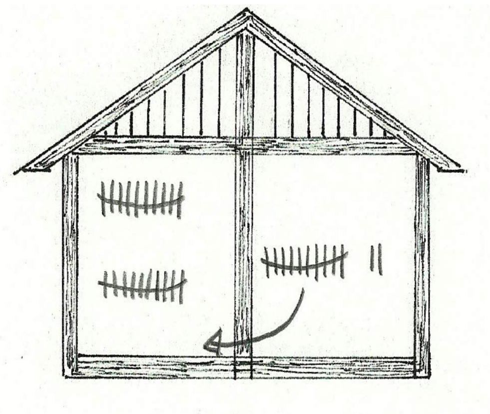
That day he went out into the forest, and struggled to pass between the heavy branches that often blocked his way. He cut as many sticks as he could, but when he was making his way home, his load was not as heavy as he was used to. When he swung his load down in front of the wooden shed, he made only 1 bundle of ten and he still had 3 loose sticks. How many sticks is that?

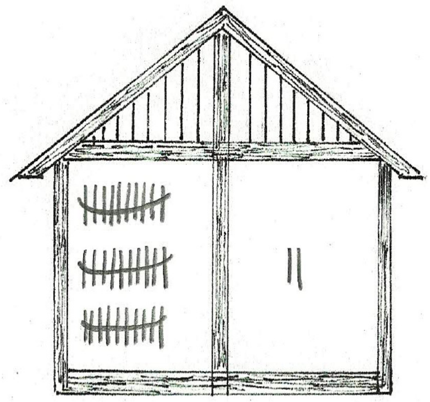
So, let us take out our counters and add up how many he had altogether now - he had 19 sticks, and now he brought another 13. The children make the correct number of bundles and count the loose sticks needed, and place the 19 in the correct 'rooms' and the 13 below:

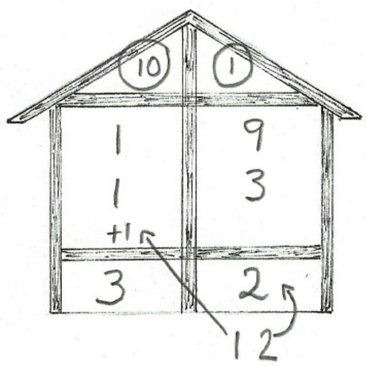


We always start with the loose sticks, you remember? Let us add the number of loose sticks together: there were how many? [9 loose sticks]. And how many new sticks?

[3]. Put them together, and how many loose sticks do we have now? [12]. What do you think the woodcutter did now, when he has 12 loose sticks? [children answer: make a bundle of 10!] Good! So make a bundle of 10 and tie it with an elastic band. What will the woodcutter do with this bundle of 10? [children answer: he will put it in the 10's room (the 'bundles' room)]. Good!
How many loose sticks will he have now? [2]. Where will he put them? [children answer: in the 1's room ('loose sticks' room)].
Now how many bundles and how many sticks does the woodcutter have now? [3 bundles of 10 sticks, and 2 loose sticks]. And how many sticks is that altogether? [32]
Having done the carrying sum with the counters, the teacher now shows the children how we do the sum in writing. Carrying the bundle of 10 across to the 'bundles' room is the important new learning that the children need to understand:

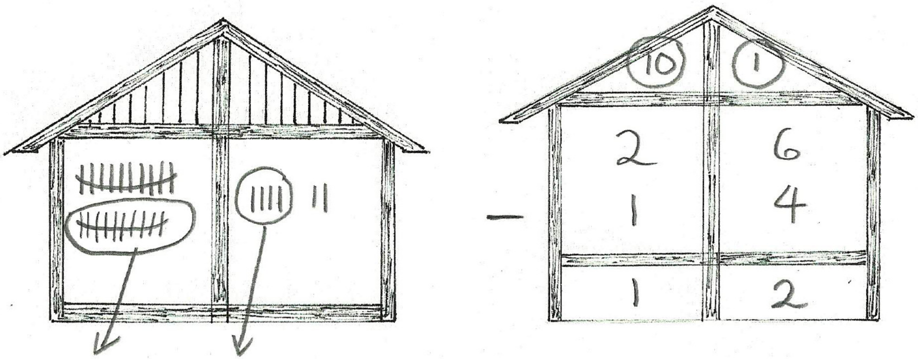




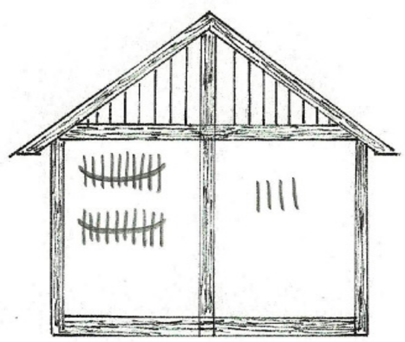


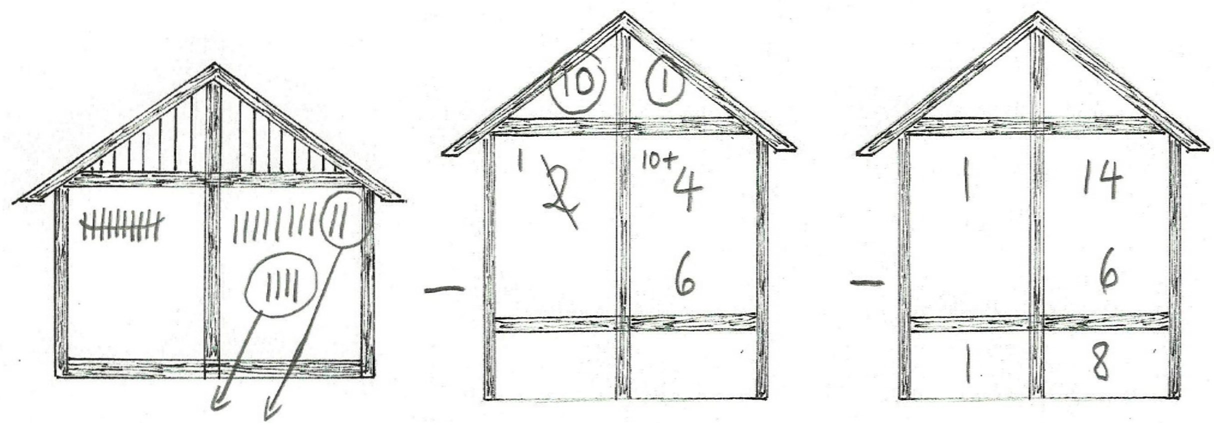
....
When addition has been firmly established through practising many different story-sums, vertical subtraction can be introduced, again using the woodcutter's story:
We have learnt a lot about how the woodcutter stores the sticks every day when he brings them from the forest to his wooden shed. Of course, he also sells his sticks to the many people who need them to build their fences, gates, wooden huts, walls and so on.
And so, at the end of the day, the village people know that the woodcutter is back from doing his hard work in the forest, and they come to him to buy the number of sticks they need.

The woodcutter always greeted everyone very heartily, and was happy they came to buy sticks from him. One day, there were 26 sticks altogether in his store. How many bundles of 10 were there? And how many loose sticks?
"Please, Mr. Woodcutter," said a builder, "I need 14 strong sticks for the small house I am building down in the village. Have you got enough to sell me 14 sticks?"
"Of course," said the woodcutter, "I always know exactly how many sticks I have in my store! I have 2 bundles of 10 and 6 loose sticks, and that makes 26 sticks altogether."
And so, the woodcutter went to fetch the 14 strong sticks the builder had asked for.
Now, children, let's all take out 26 sticks and arrange them in the two rooms, the way the woodcutter did it every day.
Now what will the woodcutter do, to give the builder his 14 sticks?
The children now work out that the woodcutter will take 4 loose sticks out of the 1's room, and one bundle of 10 out of the 10's room.
The teacher then shows the children how this vertical sum is written. *Which sign will we be using, if the woodcutter <u>gives</u> 14 sticks to the builder? [Minus sign].*



Day after day many examples of subtraction story-sums are worked through, without carrying, until the children understand subtraction well.
....
When the children are ready for the next step, subtraction sums with carrying can be introduced. Now the story-sum will involve taking a bundle of 10 from the 10's room in order to be able to sell the number of loose sticks the customer is asking for. For example, starting simply: the woodcutter has 24 sticks in his wooden shed, and some-one wishes to buy 6 sticks. A good teacher will <u>ask</u> the children how the woodcutter could do this, rather than telling them. The children will say the woodcutter has to carry one bundle of 10 into the 1's room, and undo this bundle to be able to give the buyer his 6 sticks. Let them do this with their counters, and then show them how we write this as a vertical sum:



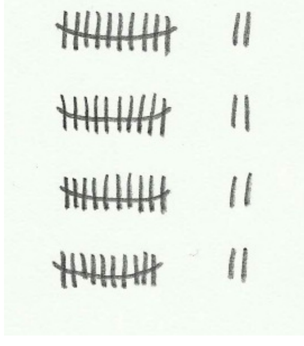


....
To introduce vertical multiplication sums, the story needs to have something that is done several times. For example:

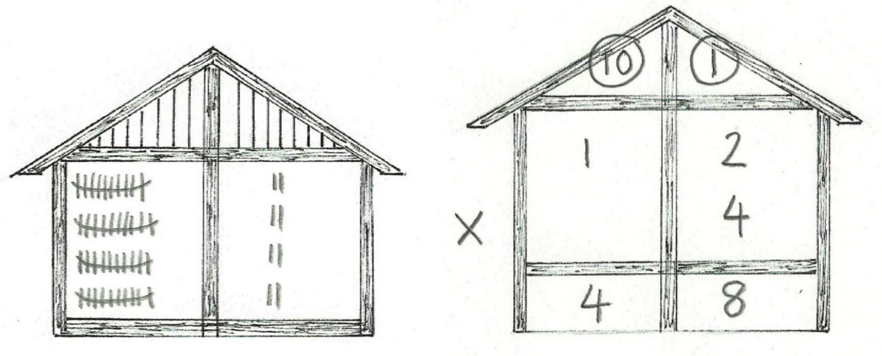
The woodcutter needed some help in keeping enough sticks in his wooden shed, as many people came to buy them. And so, the woodcutter asked a friend of his to cut sticks for him. This friend liked to cut 12 sticks at a time, and tie them into a pile. As he was not as strong as the woodcutter, he could only carry a pile of 12 sticks on his back.

When he had cut 4 piles, he wanted to bring them to the woodcutter. How many sticks were in each pile? [12] How many times did he have to walk to the woodcutter to give him these piles? [4 times].

Let's work out how many sticks this is, and how the woodcutter counted them: so, take out your counters and make 4 piles of 12 sticks each. What did the woodcutter do to each pile? [made 1 bundle of 10 and had 2 loose sticks left over - the children do this with their sticks].

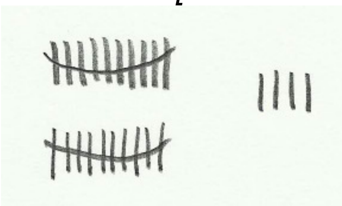


How many times did he have 2 loose sticks? [4 times]. 4 x 2 loose sticks is how many sticks altogether? [8 loose sticks]. Where did he put these sticks? [in the 1's room - children place the 8 loose sticks in the 1's room (right hand side)]
How many times did he have the bundles of 10? [4 times]. How many sticks in 4 bundles of 10? [40]. Where did he place the bundles? [in the 10's room - children place the 4 bundles in the 10's room (left hand side)]. How many sticks did the friend cut altogether? [48].

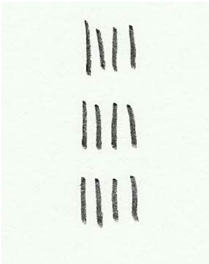


Many similar sums can be done to consolidate their understanding of vertical multiplication sums, <u>without</u> carrying (for example 3 x 11; 2 x 14; 3 x 23; 2 x 42; 4 x 32) each time with a new story.
....
Once this is well understood, carrying could be introduced, unless the teacher feels that it is better to leave this for the next year. A story-sum is needed to make the multiplication sum meaningful and clear, for example:
One of the builders in the village wanted to build a new house on the edge of the forest, and needed many sticks. This house was to be a magnificent house, with a high roof, and lots of wooden sticks to make it look like it was part of the forest.
He needed so many sticks, that the woodcutter realised he would not be able to keep up with the number of sticks the builder wanted every day. And so, he decided to take his two donkeys and cart, go to the neighbouring village to buy more sticks.
The cart went hobbledy-hobbledy over the rough roads, while the donkeys were quite used to finding their way between the stones and holes on the road.
When they got to the neighbouring village, the woodcutter gave his donkeys some food to eat, while he went to talk with the woodcutter of that village. This man sold his wood in piles of 24 (24 sticks in a pile at a time).
The woodcutter went to his cart to see how many piles of 24 sticks he would be able to fit on his cart. Mmm, he thought, I am sure I can carry 3 piles of 24 sticks on my cart. And so he asked for 3 piles from the neighbouring village's woodcutter, paid for them, and loaded them on his cart.
It was along and dusty road travelling back to his home. The two donkeys knew they were going home now, so they trotted along faster than before, knowing that soon they would be grazing in their favourite fields.
When the woodcutter came to his home, he placed the three piles of sticks on the ground in front of his wooden shed. Each pile, as we know, has 24 sticks in it. The children count out 3 piles of 24 sticks and place them in front of them.

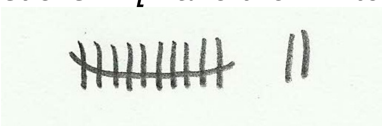
As usual, the woodcutter wanted to make bundles of 10 so that he could store them in the 10's room. The children now do this with their counters.
Well, children, how many bundles of 10 could the woodcutter make from each pile of 24 sticks? [2 bundles of 10]. And how many loose sticks left over? [4].



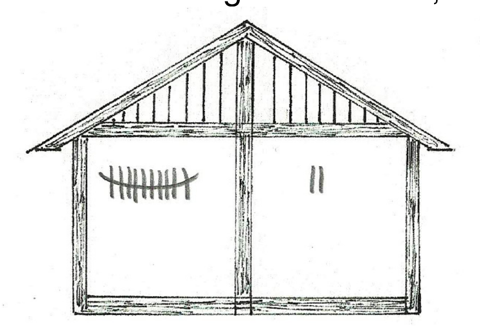
How many piles did the woodcutter bring home? [3]. So, how many <u>times</u> did the woodcutter have 4 loose sticks? [3 <u>times</u>]. Put the <u>3 times</u> 4 loose sticks together, with a small space between each group:



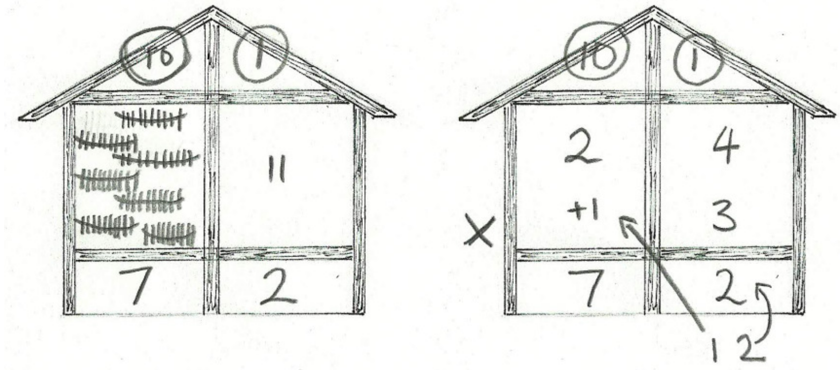
How many loose sticks are there now? [12]. What will the woodcutter do if he has 12 sticks? [make them into 1 bundle of 10 and leave 2 loose sticks].



Where did he put the 2 loose sticks? [in the 1's room]. And what did he do with the bundle of 10? [he put the bundle into the 10's room]. The children place the 2 loose sticks on the right hand side, and the bundle of 10 on the left hand side.



And now, what about the rest of the piles? Each pile has how many bundles of 10? [2 bundles]. How many piles did the woodcutter bring home? [3 piles]. So how many <u>times</u> did the woodcutter have 2 bundles? [3 <u>times</u>]. How bundles is that altogether? - Look at your counters! [6 bundles of 10]. Place them in the 10's room!
Now the teacher shows the class how we write this sum on the board, emphasising <u>how</u> the carrying is written:

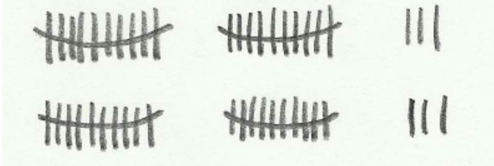


Multiplication sums with carrying will need a lot of practice, keeping on emphasising how the carrying is done.

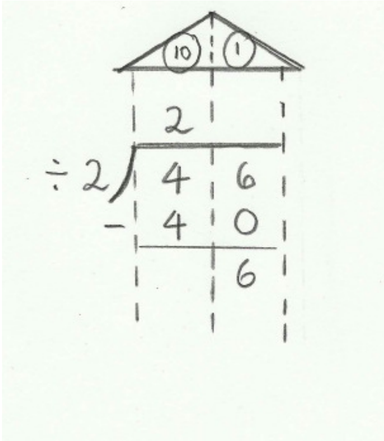
....

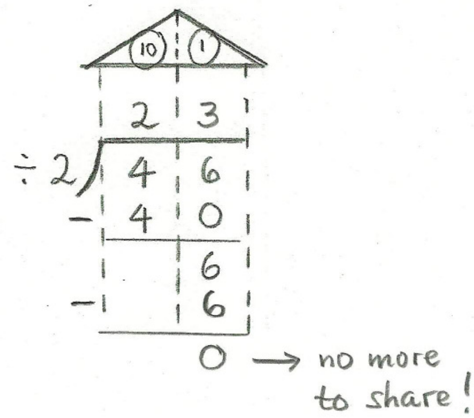
Simple vertical division could be introduced, to plant the idea for learning division with carrying in later years. The teacher may decide to leave this introduction to the next year, if she or he feels that the children are not ready to understand this yet. Again, it is important to introduce this with a story-sum, for example;

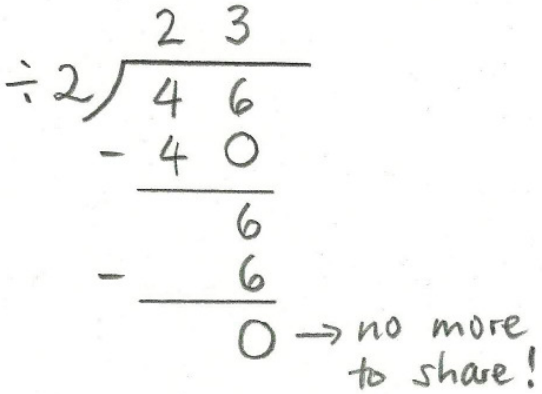
One day, the woodcutter had a look in his wooden shed to see how many sticks he had. He found there were 4 bundles of 10 and 6 loose sticks. How many sticks is that altogether? [46].
He had just finished counting his sticks, when two men arrived at his shed. They asked the woodcutter, "how many sticks do you have to sell us today?" "Well," replied the woodcutter, "I have 46 sticks altogether." The two men looked a little disappointed, and said, "we had hoped you could give us more than that." "I am sorry," said the woodcutter, "but tomorrow I will have more again."
"Good!" said the men, "but we would like to share what you have now equally between us, so we can start our work today, and we will be back tomorrow to ask you for more sticks.
So the woodcutter had to share the 46 sticks he had between the two men. Let us take out our counters, and prepare 46 sticks in our cardboard sheds.
The children place 4 bundles of 10 and 6 loose sticks in their cardboard sheds.
How will the woodcutter share out the 46 sticks between the 2 men?
Allow the children to suggest ways in which they could share out the sticks. Ask the children to first share the bundles and then the loose sticks, as this is what we always need to do when sharing (the reasons need not be given at this time, as this will become clear when they do vertical division with carrying in later years). Make sure all the children share the counters correctly, by placing them as follows:



Now we can lead the children in how to write what we have done in a vertical division sum:







The important thing here is to remind them about place value: in 46, the '4' stands for 4 tens, and how much is 40 shared between 2 men? [20]. The '6' stands for 6 loose sticks, and how many sticks will each man get when we share them equally? [3 each]
As before, this new way of doing division sums without carrying will need to be practised - always using a story-sum, followed by working with counters - for several days for children to understand how to do it with confidence.

THE COMMUTATIVE LAW

The commutative law says that 3 x 2 is the same as 2 x 3.
We need to bring this understanding to the children by creating a little story-sum in which a character does something 3 times, each time carrying 2 items. We let them count out their counters, showing 3 groups of 2 items. We ask them how to write this sum on the board. [3 x 2 = 6]
Then we can ask them, but what would happen if the character in the story did the same task 2 times, each time carrying 3 items. We let them to place their counters according to the new way of doing the task, showing 2 groups of 3 items. Again, we ask them how to write this sum on the board. [2 x 3 = 6]
We can then ask them whether it mattered whether the character did the task 3 times, each time carrying 2 items, or whether the task was done 2 times, each time carrying 3 items.

From this they can discover that it comes to the same thing. If you were the character, what would you do - rather walk 3 times, carrying a lighter load, or 2 times, carrying a heavier load? This can lead to a lively discussion in the class, helping to consolidate the understanding of the commutative law.
We can repeat this work with the counters, using different numbers
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THE NUMBER '0'

The children will have learnt that the number ‘0’ means ‘none’ or ‘nothing’ when they learnt that in 10 there is one ‘ten’ and no ‘units’.
We now need to introduce the number ‘0’ in the four operations, emphasising in each case that this number says ‘nothing’, ‘no times’ or ‘none’.
In addition, we can say if we have 3 apples, and we add ‘no’ apples, how many apples do we still have? [3 apples]:

$$3 + 0 = 3$$

In subtraction, if we have 5 apples, and we give away ‘no’ apples, how many apples do we still have? [5 apples]:

$$5 – 0 = 5$$

In multiplication, we can say:

If I go **3 times** to fetch 4 apples, how many apples do I have altogether? [12 apples altogether]:

$$3 \times 4 = 12$$

If I go **2 times** to fetch 4 apples, how many apples do I have altogether? [8 apples altogether]:

$$2 \times 4 = 8$$

If I go **1 time** to fetch 4 apples, how many apples do I have altogether? [4 apples altogether]:

$$1 \times 4 = 4$$

But if I go **no times** to fetch 4 apples, how many apples do I have? [no apples!]:

$$0 \times 4 = 0$$

The same is true the other way round:

If I go 3 times to fetch **2 apples**, how many apples do I have altogether? [6 apples altogether]:

$$3 \times 2 = 6$$

If I go 3 times to fetch **1 apple**, how many apples do I have altogether? [3 apples altogether]:

$$3 \times 1 = 3$$

If I go 3 times to fetch **no apples**, how many apples do I have? [no apples!]:

$$3 \times 0 = 0$$

To make the above real for the child, we can create a little story-sum for the first way, and a different story-sum (for the following day) for the second way. In both cases it is good to do the story-sums with counters, which makes it clear that ‘no times’ and ‘no apples’ means we have no apples at all.

In division, we can do something similar:

If we share 6 apples between 3 people, how many will each person get? [2 apples each]

$6 \div 3 = 2$

If we share 3 apples between 3 people, how many will each person get? [1 apple each]

$3 \div 3 = 1$

If we share **no apples** between 3 people, how many will each person get? [no apples!!!]

$0 \div 3 = 0$

And, the other way around:

If we have 4 apples, and we **share** them between 2 people, how many will each person get? [2 apples each]

$4 \div \mathbf{2} = 2$

If we do **not** **share** 4 apples, what happens? Does anyone get anything? [no one gets any apples]

$4 \div \mathbf{0} = 0$

Again, as with multiplication, to make the above real for the child, we can create a little story-sum for the first way, and a different story-sum (for the following day) for the second way. In both cases it is good to do the story-sums with counters, which makes it clear that if there are 'no apples' to be shared and when there is 'no sharing' it means we do not receive any apples at all.

NUMBER PATTERNS

To generate interest in numbers, we can explore odd and even numbers, asking the children to find out what is special about each; what happens when you add, subtract, multiply or divide them. This will encourage children to be inquisitive, to find things out for themselves.